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Developing Specialized Content Knowledge for Equitable Practice in Mathematics: Exploring a Constructivist Approach to Preservice Teacher Education

Jane Rector Wilkes

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Developing Specialized Content Knowledge for Equitable Practice in Mathematics:
Exploring a Constructivist Approach to Preservice Teacher Education

by

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Dedication

To the memory of my mother and father,
who gave me a wonderful childhood, precious memories,
and the encouragement and motivation to last me a lifetime.
They would be so proud of this accomplishment.

Acknowledgements

First, I would like to thank my Lord and Savior Jesus Christ, without whose help I could not have obtained this degree. Philippians 4:13 says, “I can do all things through Christ who strengthens me.” This verse sustained me throughout my study and continues to be a source of guidance in all aspects of my daily life.

My husband Buddy has been supportive and encouraging from the beginning of this process, and for that I am very grateful. He has taken on my share of household duties as well, so that I could work on my research. I am so blessed to have him as my partner in life.

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My sister Betsy has read all my work and helped me to refine it chapter by chapter. I could not have done it without her help. I am also appreciative of the moral

support I have received from my sisters, Tomi and Harriet, and my friends, Cindy and Vickie.

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Abstract

A successful teacher education program prepares preservice teachers to provide high quality mathematics education for all students. In order to effectively address the needs of diverse and historically underserved groups of students, future teachers need to have a deep understanding of both basic content knowledge and pedagogical content knowledge. The purpose of this study was to examine ways to support preservice teachers in improving procedural and conceptual content mastery, as well as the specialized content knowledge that they will need in order to feel empowered to teach their future students.

Based on the theoretical frameworks of two components of mathematical knowledge for teaching, common content knowledge (CCK) and specialized content knowledge (SCK) (Ball, Thames, & Phelps, 2008), the combined components of reformed pedagogy (Smith, 2013), and culturally responsive pedagogy (Ladson-Billings, 2009), I enacted a three-phase, intervention-based, action research. Twenty-seven preservice teachers participated in this study, completing all three stages of the intervention. The three stages focused on 1) increasing their common content knowledge, 2) developing their specialized content knowledge, and 3) providing an opportunity to practice teach with their peers.

Qualitative analysis revealed positive growth in both common content knowledge and specialized content knowledge. Data also indicates that the preservice teachers grew

to appreciate the value of teaching for conceptual understanding instead of teaching exclusively for procedural understanding. Findings indicate that preservice teachers became adept at defining and evaluating the specialized content knowledge of other teachers but needed additional support for demonstrating this in their own teaching practice. Implications for teachers, teacher educators and others who provide instructional support to teachers are discussed.

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Chapter 1

Introduction

I suspect every teacher who has worked in a classroom of children for any significant amount of time has experienced, both vicariously and personally, the “Aha moment.” It is that moment when “the light goes on,” and the students or the teacher themselves find a new or more complete understanding of something in the classroom. It is the instant when understanding occurs and the student begins to move forward more confidently. For the learner, a teacher who can inspire these moments is accessing one of the most powerful tools in instruction, a tool capable of generating intrinsic motivation in the learner. For a new teacher, finding effective strategies that can do this is difficult, and without a deep, conceptual understanding of the content that is to be taught, it is near impossible.

In my own practice as a college math instructor for elementary preservice teachers (PSTs), one of my more effective lessons on the division of fractions often leads to this powerful learning experience. In this lesson, I pose the problem “ $4/15 \div 2/3$ ” to my class of PSTs. I then show them an example of student work that shows, “ $4 \div 2 = 2$ and $15 \div 3 = 5$, so the answer must be $2/5$.” I then ask the PSTs, “How would you respond to this student’s solution?” The most common response is to tell the student that their method would not work, and that the procedure of inverting and multiplying would be necessary.

They also typically suggest that the student was incorrectly confusing the algorithm for multiplying fractions with that of dividing fractions. I then ask the PSTs to find the correct answer themselves and share their work with the class. Now, the student response does arrive at the correct answer. This is certain. Yet, most PSTs are amazed that their method produced the same answer as the student's answer. Feigning my own surprise, I then suggest that we try another problem. Students remain in disbelief of the results even after multiple different examples proved the legitimacy of the student's method. This situation is a clear example of the common problem in math teacher education. PSTs often know only one procedure for solving a math problem, likely due to the emphasis on algorithms and rote memorization in their own early math education. They are amazed by this "Aha moment" in the lesson and also curious as to why they had never known this mathematical concept of division until this point.

In my role as a professor at a small, rural university in the southeastern United States, the undergraduate prospective teachers with whom I work typically go to the nearby communities to begin their careers in education after graduation. The schools in this area have historically been underperforming and have earned the nickname "the corridor of shame" (Findlay, 2017, p. 1). It is well documented that for many years now, the rural school districts along this stretch of Interstate 95 have failed to provide even a minimally adequate education to the mostly poor African American and Caucasian students they serve. Some researchers have drawn a direct connection between this inequitable and ineffective educational system and the cycle of poverty these residents have been stuck in for generations (Knight, 2019). The PSTs in my classroom today will be the teachers in these classrooms tomorrow. Without a strong conceptual understanding

of mathematics and the subsequent proficiency in the instructional strategies that make conceptual understanding accessible, these future teachers will likely only perpetuate this problem. With this problem of practice in mind, I have focused this dissertation on becoming more familiar with the theories that guide and the strategies associated with teaching for conceptual understanding in preservice teacher education. A second aim of this study is to develop the intrinsic interest in and ability to teach for conceptual understanding among of PSTs in the hopes of breaking the cycle of inequitable and ineffective educational practices in our university's surrounding communities.

In this first chapter, I will provide an overview of the common indicators associated with a lack of conceptual understanding among PSTs in early math education and how research has shown that this has a direct impact on student learning. I will also provide an overview of the literature that theorizes and applies the strategies associated with teaching for conceptual understanding in mathematics. After establishing a theoretical framework for the significance of the problem, how it guides the design of this study, and the potential strategies for its remediation, I will provide a brief summary of the research design and positionality of this study and how it will engage both my students and me in the process of reflection and learning through action research.

Problem of Practice

Teachers learn to teach primarily from their own learning experiences (Darling-Hammond, 2000). These experiences have often been solely replication of procedures and have not empowered teachers with a deep understanding of concepts (Darling-Hammond, 2000). This lack of empowerment often leads to anxiety about teaching

certain concepts (Rayner, Pitsolanis, & Osana, 2009) such as fractions. For example, teaching mathematics often requires multiplication and division of fractions. Research indicates that mathematics teachers often cannot explain their thinking, cannot draw diagrams to match their algorithms, and cannot decide on appropriate operations to use during problem solving activities (Izsak, Jacobson, de Araujo, & Orrill, 2012). Studies over the last 25 years have shown that the mathematical content knowledge of many teachers is “dismayingly thin” (Ball, Hill, and Bass, 2005, p. 14).

In their study, Marshman and Porter (2013) concluded that most of the pre-service participants did not have the necessary depth of content knowledge to diagnose student misconceptions and to provide appropriate feedback. The researchers further suggested that PSTs were not able to acknowledge the students’ levels of understanding because they were at the same level of understanding as the students (Marshman & Porter, 2013). In this situation, teachers with limited conceptual understanding of the content they need to teach often rely heavily on textbooks for explanations and examples and assign students to work individually on worksheets (Sutton & Krueger, 2002). These practices portray math as a set of facts and procedures, thus making a deep understanding of math concepts difficult for the students they teach (Sutton & Krueger, 2002). Furthermore, these less knowledgeable, and therefore less effective, teachers will more than likely be hired and stay in lower paying, high-poverty school districts (Scoppe, 2017). These districts often have a higher turnover and attrition rate compared to their more affluent counterparts (Garcia & Weiss, 2019).

The literature confirms the necessity of teacher education programs to increase the support for PSTs in building their own common content knowledge (CCK) and in

building rich, supportive learning environments that meet the needs of diverse populations of students. South Carolina is a prime location for an action research study that has the potential to increase the number of high- quality teachers in the state. These future teachers will serve students in diverse areas, including the high poverty areas, where students do not always receive equitable educational opportunities. The setting for this study is in a South Carolina university where I teach math content for elementary PSTs and will be guided by the theories discussed in the theoretical framework.

Theoretical Framework

Successful teachers not only possess CCK and specialized content knowledge (SCK), but they also know that meaningful mathematics lessons occur when students can relate the lesson content to their own backgrounds (Wlodkowski & Ginsberg, 1995). In recent decades, research has shown that conceptual knowledge plays an important role in the knowledge and activity of teachers who are proficient in both of these areas of knowledge (National Council of Teachers of Mathematics [NCTM], 2000). This study examines strategies of reformed pedagogical practices (Smith, 2013) to develop the knowledge that PSTs need in order to become successful teachers. Given the highly specific context in which this study takes place and its inherent connection to issues of educational inequality, elements of culturally responsive pedagogy (Ladson-Billings, 2009) played a critical role in the decisions made during this study.

The idea of a reformed pedagogy (Smith, 2013) that prescribes active learning for teaching conceptual understanding in mathematics rather than an exclusively procedural understanding is a primary theory guiding this study. As such, Smith (2013) defined

reformed pedagogy in mathematics as the change in teaching practice from the promotion of procedural understanding to the promotion of conceptual understanding. This shift from more traditional teaching methods of mathematics is based on theories that highlight the importance of creating a productive learning environment, using interdisciplinary teaching, giving the child opportunities for input, and using developmental materials to teach (Eichelberger, 2011). Instructional strategies for building conceptual understanding often involve tactile learning styles that incorporate the use of concrete objects, or manipulatives, to begin developmentally appropriate new learning of mathematical concepts (Beckmann, 2014). The effective models of reformed pedagogy are student-centered and are implemented through high student engagement, collaboration, and metacognition (Smith & Mancy, 2018).

The contributions of Shulman (1986), who first introduced the term pedagogical content knowledge (PCK) as the “missing paradigm” in research on teaching and teacher knowledge, brought attention to the “blind spot” that characterized most teacher research, state-level programs of teacher evaluation, and teacher certification. Building on this premise in mathematics education, research by Ball, Thames, and Phelps (2008) sought to refine the concepts of PCK and give educators a better explanation of how this knowledge is used in teaching effectively. These efforts led to the development of the mathematical knowledge for teaching (MKT) framework (Ball et al., 2008), which provides a more specific theory for pedagogical content knowledge (PCK) in mathematics education. The MKT framework includes six categories: three categories for subject matter knowledge and three categories for pedagogical content knowledge. One

category, CCK, from the subject matter knowledge, and one category, SCK, from the pedagogical content knowledge guide the development of this study (Ball et al., 2008).

Hill, Ball, and Shilling (2008) defined CCK “knowledge that is used in the work of teaching in ways in common with how it is used in any other professions or occupations that also use mathematics (p. 377)”. Accountants, engineers, actuaries, and countless others in math-related professions use the common content knowledge of algorithms and formulas to perform computations related to their work. Teachers of mathematics must obviously have mathematical content knowledge in order to impart the knowledge to others. Being able to use the curriculum effectively and work with standards depends on the teacher’s knowledge of the subject matter (Ball, 2003).

Ball, Thames, and Phelps (2008) defined SCK as the mathematical knowledge that is necessary specifically for teachers and is concerned with the demands of teaching, such as representing meaning in mathematical concepts and using mathematical reasoning and insight. Teachers need to be able to identify patterns in student errors and determine if different nonstandard approaches are valid. SCK gives teachers the necessary tools to effectively explain concepts and represent them with drawings and diagrams, and to give meaningful examples to make a specific mathematical point (Ball et al., 2008).

Culturally responsive pedagogy is a term created by Gloria Ladson-Billings (1994) to describe “a pedagogy that empowers students intellectually, emotionally, and politically by using cultural reference to impart knowledge, skills, and attitude.” (p. 382)

Culturally responsive pedagogy is a pedagogy grounded in teachers displaying cultural

competence and enabling each student to experience the course content in her cultural context (Ladson-Billings, 2009). According to Wlodkowski and Ginsberg (1995), the four conditions necessary for culturally responsive teaching include *inclusion*, which is learned through student collaboration and cooperative learning; *positive attitudes*, which is accomplished through problem solving models, attention to multiple styles, and experiential learning; *enhanced meaning*, which is accomplished through problem solving and relevant experiences; and *engendering competence* by using multiple types of assessment and encouraging self-assessment.

The elements of reformed pedagogy and culturally responsive pedagogy both support the development of CCK and SCK for PSTs through learning experiences that are relevant and meaningful to their prior experiences. Both theories also give PSTs a way of reflecting and refining ideas through the common component of class collaboration. As PSTs begin to embrace the ideas of reformed pedagogical and culturally responsive practices, they will develop an intrinsic motivation for teaching through conceptual understanding and using these same practices in their future classrooms. The cyclical nature of the process gives hope for producing more highly qualified teachers, who in turn will more effectively teach mathematics to future generations of diverse learners.

Research Questions

It is the purpose of this study to examine ways to support my student PSTs in improving procedural and conceptual math content mastery, as well as the specialized content knowledge they will need in order to feel empowered to teach their future

students. The evidence collected through results of skills assessments in my classes has led me to choose multiplication and division of fractions as a focus for this study.

Teacher beliefs about the efficacy of conceptual methods of teaching mathematics versus procedural teaching methods need to be developed.

The intervention method consisted of a three-phase plan. During the first phase, the PSTs experienced pedagogy as learners, and assessments focused on CCK. Reflecting on pedagogy was the focus of the second stage of the plan, as PSTs began the transition from learners to teachers. In the last phase of the intervention, the PSTs practiced pedagogy as they planned and implemented their own lessons. During the last phase, all students had opportunities to participate as teachers, learners, and as evaluators in a rotation of the three assignments.

The research question for my study was:

1. What are the important factors to consider when developing instructional strategies that promote specialized content knowledge and the intrinsic motivation to teach for conceptual understanding among preservice elementary mathematics teachers?

I selected this question based on the nature of the problem of practice and the theories I planned to use during the enactment and study of the intervention. The question was focused on the specific aspects of my practice and the learning I hoped to facilitate in my students. Support for developing both CCK and SCK were aspects of the preservice program that warranted more attention. My colleagues in the university's math content teacher program and I had discussed the need for increasing the conceptual understanding

of the mathematics we teach to provide a proficient background in mathematics teaching and learning. It was critical that I examined my positionality and reflected on the potential that my teaching experiences had on forming my research and swaying my analysis of what occurred during the research process.

Positionality in Action Research

In almost all forms of traditional qualitative research, the positionality of the researcher is an important element of the work (Creswell, 2014). Herr and Anderson (2015) defined positionality as the relationship of a researcher her setting and participants. The importance of positionality in an action research study cannot be overstated, since the researcher's positionality can shape the research and influence her interpretation of the research topic. In action research, an insider who does a study will have access to "the truth" only as she sees it, although the researcher truth, which is affected by her positionality, is only one among many (Herr & Anderson, 2015). A positionality statement tells who the researcher is, the relationship of the researcher to the research study, and how she views the world (Holmes, 2010). To this end, I am sharing a statement here about my background and personal stance towards mathematics education. In doing so, I identify how I play various roles at different times in this study, each role having specific connections to my positionality and I how I must respond in different situations that occur during the study.

I have lived my entire life in a small, rural town in South Carolina. During my school years, the town thrived with the many textile mills that employed most of the citizens in the county. My father owned a real estate and insurance business. A maid and

cook took care of our house, while my mother chauffeured and entertained my siblings and me. We were privileged, but we did not know it at the time. Our parents encouraged us and praised us. They valued an education and expected nothing less than our best efforts and behavior at school. I had a love for math, enjoyed doing homework and taking tests in the subject, and found it somewhat analogous to working crossword puzzles and cryptograms. I saw the relevance of the content through my father's explanations and discussions, not through the lessons in the classroom.

I wanted to teach math and share my love for the subject and for learning in general. I taught in a high school math classroom at my alma mater for 28 years, and never lost the passion for teaching. The mills closed and along with the loss of jobs came the loss of most of the middle class. Demographics changed, and our town lost its tax base. The poverty index in the school district rose, so that our schools became Title I identified based on the large number of students eligible for free and reduced lunch. Opportunities for students to take educational field trips and to have learning experiences through guest artists and consultants were limited. Funding for facilities and instructional resources were cut, forcing teachers to become more creative with finding hands-on instructional materials. I used homemade materials to help students problem solve and complete conceptually based activities. I was determined not to use exclusively lecture and worksheets, as I came to understand that students found these methods to be painfully tedious.

After 28 years in the classroom, I became a teacher specialist for the South Carolina State Department of Education with the job of helping teachers and building capacity in underperforming schools. Ironically, I was assigned to my hometown school

district, where I worked with every math teacher in the district in Grades K–12 to develop and implement appropriate and effective lessons. After 33 years working in this school district, I took a position in the math department at a small, rural university very close to my hometown. Soon I was assigned more duties with math content for PSTs, becoming a liaison between the Department of Mathematics and the College of Education at the university.

I now have the opportunity to instill a love for learning and developing a deep conceptual understanding of math concepts in those who will touch so many students in future generations. The positive impact of the instructional strategies that I choose to use has exponential potential for the future. This research study will help me to make the best choices in my own practice.

Opportunities to help my own district are still available, as I am serving my third four-year term on the school board of trustees for the district. Through this position, I have gained a better perspective of the funding issues that affect school resource allocations. My town benefitted from the Abbeville County School District versus the State of South Carolina, not because it is located along the I-95 corridor but because the poverty index is above 80% (82%). At biweekly meetings, I am informed of instructional progress in the district, and I have input into decisions affecting curriculum, instruction, programs, initiatives, and teacher recruitment and retention.

My positionality as a woman has made me aware of the stereotypes of women in math-related fields and of the anxieties that some female students face in math classes. I am also aware of my White privilege and the benefits that come with my skin color

(hooks, 1994), something that was not entirely obvious to me until late adulthood. Dyconscious racism is tacitly accepting dominant White norms and privileges (King, 1991). I fall into this category along with many other well-intentioned White people. Not completely understanding White privilege, or even trying to understand it, has been my option. That in itself is a White privilege. As a teacher of 42 years, I have always been sensitive to the needs of all children as individuals, and I have enjoyed excellent relationships with students and parents of diverse backgrounds. My entire life I have been a nurturer, and I firmly believe that a caring pedagogy is essential for effective teaching. As a teacher of PSTs, I have a responsibility to address the present state of educational inequities and to increase the awareness of these future teachers of their own biases and prejudices. Until teachers realize their biases, they cannot confront them and take the steps to make necessary changes. I believe that children should see themselves “in mirrors and through windows” (Style, 1996, p. 3) just as much in math class as they do in literature.

I collaborated with other professors who teach classes in the math teacher course sequence in order to help me reflect on lessons and results, and in order to help with identification of researcher bias. Collaboration with participants included member checking to ensure accuracy of findings. My positionality as the teacher of the class had the potential to lead to tainted qualitative data. Some students may have felt that they should answer questions about their feelings and strategy preferences with answers that they felt the teacher wanted to hear. Assuring students that their honest answers would be much more helpful to the study decreased the chances of inaccurate student responses.

Member checking was a way to assure participants that their perspectives would be reported in an honest and accurate manner (Efron & Ravid, 2013).

Research Design

Action research is the most appropriate type inquiry for my study. The definition of action research includes inquiry done by an insider to an organization or a community and is oriented toward some cycle of actions to address a particular problematic situation (Herr & Anderson, 2015). Action research demands an intervention and constitutes a spiral of cycles of planning, implementing, observing, and reflecting (Herr & Anderson, 2015), which is evident in my three- phase plan for research.

I chose a qualitative case study design (Creswell, 2014) for this study as a means of discovering methods to improve my own practice in developing quality preservice support for future elementary math teachers. Qualitative case study typically involves an in-depth exploration of a program, event, activity, or process, or one or more individuals, and is a common approach used in educational action research (Efron & Ravid, 2013). This particular study explored the teaching practices in a PST class at a small university setting in South Carolina. The intent of the research study was to find a solution to an authentic problem of how to improve CCK and SCK to support these prospective teachers in the understanding of fraction meaning and the operations of multiplication and division on fractions.

In past assessments of PSTs, the meaning and operations on fractions had been historically the biggest areas of weakness. I purposefully chose the participants, PSTs in the second course of the required math content sequence for elementary teachers, for this

study because I would have an advantage of action research, which is familiarity with participants in the study setting (Efron & Ravid, 2013). I selected this class for the research study because the specific content topics in this course on rational numbers allow me to support them in developing CCK of fractions. The students in the class were majority Caucasian and female, although both genders were represented in the class, as were other ethnic groups, including African American, Hispanic, and Asian students.

Although many of the students shared some common physical attributes, the variety of cultural and prior educational experiences that students brought with them gave rise to a wide range of beliefs and attitudes about teaching and learning among the class. To learn the meaning that these participants held about the problem of the study, I chose multiple data collection instruments to capture useful information and to help me in the to ensure the validity of the study through triangulation. The data collection instruments included a pretest and a formative assessment to assess growth in the CCK of the students during Phase 1 of the intervention. During this phase, PSTs participated as learners, while I assumed the role as the teacher. During the initial phase, there was also evidence gathered for first impressions of the PSTs' SCK. This was done through observations, discussions, and exit slips. I used a journal to record reflections before the intervention began and after each phase of the intervention was completed. Daily teacher notes guided these reflections. During Phase 2 of the intervention, more exit slips and student reflections were utilized to gather evidence, as PSTs were in an intervention stage of reflecting on the practices of others. During the third stage, PSTs planned and implemented their own 15-minute lessons, and played the roles of learners, and “critical friends” at various times. The critical friends evaluated the teaching of the lessons with a

teacher-made rubric, and debriefing sessions were held after each round of teaching. Observations, daily exit slips, a teacher reflection journal, lesson plans, and videos were all used as multiple sources to ensure rich information for teacher and student reflection as to the efficacy of and beliefs about the reformed pedagogy of conceptual teaching methods. Once I had collected the data, I began the process of analyzing it to make meaning of the different pieces of information.

I originally used a priori coding of PST artifacts using a researcher/practitioner-designed rubric to analyze the effectiveness of the instruction for CCK and SCK development. I used a rubric based on the different components of quality teaching using SCK that was developed by Deborah Ball (1990). I used this same rubric to assess how responses in lesson plans and implementation support SCK. As I collected the data, I discovered that by developing an inductive rubric based on the performances of my different groups of students, I would have not only a way of assessing the student lessons but that I would also have a way of improving my own SCK. I also utilized thematic coding strategies to condense data into categories to help in the descriptive writing of the major ideas that emerged from student observations and journals and from researcher field notes. I will discuss these aspects of the research in greater detail in Chapter 4.

Significance of the Study

The purpose of action research is to generate knowledge to address the immediate needs of people in specific settings and also to improve practice (Herr & Anderson, 2015). Although the intent of this study was to develop the CCK and SCK of students in my own classes, there was evidence to show that the problem of practice is not unique to

my classroom situation or to my school. This study was about developing the professional disposition of teachers to encourage them to be continuous learners in their classrooms and practice (Mills, 2003), something that is likely applicable across multiple educational settings.

Supporting PSTs in developing the disposition to be continuous learners is crucial to their future teaching practice. The research done by Lee Shulman (1986) and Deborah Ball (1990) has brought attention to the fact that there is a need to reform the way teachers are trained, assessed, and certified to include knowledge that is unique to the profession. Without attending to these inadequacies in teacher education and teacher certification programs, schools may be staffed with faculty who are not prepared (Ball & McDiarmid, 1988). Attention to supporting PSTs in education programs can have a huge positive impact on the quality of mathematics education that children in this state and in the nation receive (Ball & McDiarmid, 1988). This action research has the potential to affect teachers who will serve in districts of poverty, where quality teachers are in short supply and desperately needed by the students (Garcia & Weiss, 2019). Educating PSTs with the tools of CCK, SCK, reformed pedagogy, culturally responsive pedagogy, and self-confidence can decrease the achievement gap in mathematics among diverse and underserved student populations.

This research study will not produce generalizability, but there exists potential for the findings to be transferred to other educational settings.

Limitations of the Study

The limitations of the study include researcher bias toward certain conceptual methods and strategies. Being aware of these assumptions that certain methods will be more effective than others and reflecting honestly on the data as it presents itself is critical to the subjectivity of the study. Use of a reflective teacher journal for the qualitative methods of the research provides means of self-awareness to help acknowledge and disclose subjectivity and the impact it will have on the study (Efron & Ravid, 2013). Collaboration with colleagues helped me to make sense of the data and to keep these biases and preconceived notions in check.

A second limitation is that the study focuses only on the CCK and SCK of teachers working with the particular topic of fraction multiplication and division. This study does not address the issues of MKT in other areas of mathematical content or at different levels of mathematical development. A study focusing on the topics of geometry, algebra, measurement, or data analysis may have different results than the results obtained in this research study. Also, the study was limited to PSTs who were developing CCK and SCK in the teaching of elementary mathematics. The results of the study do not necessarily transfer to PSTs who need to develop CCK and SCK for middle and secondary levels of teaching mathematics.

Organization of the Dissertation

Chapter 2 focuses on a review of the literature that extends what has been discussed so far regarding the problem of developing mathematical content knowledge for teaching (Ball et al., 2008) among PSTs. It also provides an in-depth review of the

theoretical perspectives I selected and employed during the design, enactment, and study of this problem of practice. Chapter 3 discusses the qualitative case study design (Creswell, 2014) I selected for this action research study (Efron & Ravid, 2013), as well as the various methods by which I collected and analyzed data for this study. These methods included observations and participant comments from class activities and classroom discourse, along with daily exit slips, student journal entries, and teacher field notes during each phase of the study (Creswell, 2014). Chapter 4 includes the interpretations of the research study through data coding (Saldana, 2013) of journals, field notes, and exit slips, and rubric development created with criteria of quality teaching (Ball, 1993) in terms of CCK and SCK development during the intervention period. Although the analysis provided information through some quantitative measurements, I interpreted and described the data through rich descriptions of a narrative style, which are characteristic of qualitative research (Creswell, 2014).

From the analysis of the best performing group and the whole group during Phase 3 lesson presentations, I was able to develop an inductive rubric to be used in assessing the rest of the class. The development process for this rubric is fully discussed in Chapter 4. Chapter 4 also includes an interpretation of the three main codes that emerged as a result of the coding process followed from the Saldana (2013) coding manual for qualitative data. I identified and described these three main themes, mathematical representations, means of engagement, and conceptual understanding and learning.

Chapter 5 includes a discussion of the findings and the implications of the research and provides suggestions for further research. The success of this research study would be impetus for further study of the theoretical framework in other content areas of

mathematics that would improve the researcher's practices. In this chapter, there is also a discussion of my reflection of how this research study will affect my future teaching practice for supporting PSTs and for addressing the social justice theme of my study. I provided a more detailed plan that I will use for going forward in Chapter 5. I also describe in Chapter 5 the components of action research that show ongoing monitoring and implementation of the improved practices of this study as a constant practice that is cyclical in nature.

The transferability of this study to other settings is included in the Chapter 5 discussion, along with the confirmation of the validity and reliability of the study.

Key Words

Algorithm: a step-by-step process often used in mathematics for solving problems

Case study: a qualitative research design in which there is an in-depth analysis of a case, often a program, event, activity, process, or one or more individuals (Creswell, 2014)

Classroom collaboration: when groups of students work together to search for understanding, meaning, or solutions or to create an artifact or product of their learning (NCTM, 2000)

Common content knowledge: the mathematical knowledge and skill used in settings other than teaching (Ball et al., 2008)

Conceptual understanding: an understanding of more than just isolated facts and ideas. This type understanding enables one to transfer knowledge to new situations and apply it to new contexts (NCTM, 2000).

Constructivist theory: a learning theory that suggests that humans construct knowledge and meaning from their experiences.

Cooperative learning: a teaching strategy in which small teams, each with students of different levels of ability, use a variety of learning activities to improve their understanding of a subject (Billstein, Libeskind, & Lott, 2016)

Culturally responsive pedagogy: a pedagogy that recognizes the importance of including students' cultural references in all aspects of learning (Ladson-Billings, 2009)

Dyconscious racism: the limited or distorted understandings about inequity and cultural diversity (King, 1991)

Formative assessment: monitoring student progress during instruction and learning activities, which includes feedback and opportunities to improve (Green & Johnson, 2010)

Inductive rubric: a rubric developed based on inferences made from the observations of the work of others

Invert: to turn upside In the case of division of fractions, the procedure is to invert and multiply by the divisor.

Manipulatives: objects that students can touch and move in order to help them learn mathematical concepts

Mathematical knowledge for teaching (MKT): the mathematical knowledge of teaching divided into categories for subject matter knowledge and pedagogical content knowledge (Hill et al., 2008)

Preservice teacher: a student in a professional education program, designed to train teachers to formally enter the profession at a specified level of education

Priori coding: a deductive form of analysis of data, which includes predetermining the codes that are to be used in writing a description of data (Saldana, 2013)

Problem solving: solving a problem in which the solution method is not immediately known (Billstein et al., 2016)

Procedural knowledge: the knowledge of the steps required to attain various goals (Rittle-Johnson et al., 2001).

Qualitative: related to measuring something by its quality, rather than its quantity

Rational numbers: all numbers that can be written in the form a/b , where a and b are both integers, and b does not equal zero (Billstein et al., 2016)

Reformed pedagogy: in mathematics, the change in teaching practice from the promotion of procedural understanding to the promotion of conceptual understanding (Smith, 2013)

Specialized content knowledge: Mathematical knowledge and skill unique to teachers (Ball et al., 2008)

White privilege: having privileges in society that other people do not have based on the White color of one's skin.

Chapter 2

Literature Review

Effective math education is a growing concern for nations worldwide: “The globalization of markets, the spread of information technologies, and the premium being paid for workforce skills all emphasize the mounting need for proficiency in mathematics” (National Research Council [NRC], 2001, p. xiii). Improving the mathematics knowledge of children depends on opportunities for teachers as to the high-quality instruction they must deliver (Ball, 2003).

Teachers usually teach the way they were taught. For math students, this could be a serious problem and could hinder their future success in math courses (NRC, 2001). Many teachers experienced math as a set of procedures and tricks used to get answers (Will, 2017). The concepts they learned were usually taught in isolation and in no way were connected to other math concepts or prior learning experiences (Balka, Hull, & Miles, 2003) Relevance of the math lessons was not a concern and understanding of content was not key. When those students with procedural-only learning experiences became teachers, the cycle of the irrelevant, disconnected math lessons was perpetuated. Colleges of education have powerful direct influence on elementary and middle school teachers, and it is through their PST programs that mathematical education reform can break the cycle.

Although it is obvious that teachers should have the content knowledge for what they teach, that is not sufficient knowledge for them to respond well to students and to help students become proficient in mathematics (Ball, 2003). The rote memorization that comes from procedural teaching methods is not enough to prepare students for the 21st-century skills necessary for them to become productive citizens and productive workers in the future workforce. Instead of teaching isolated skills, PSTs need to learn to teach for conceptual understanding of specific math content and to actively build students' new knowledge from their prior knowledge. As of now, few preservice programs for elementary teachers emphasize the deep learning of specific math content (Ambrose, 2004). With a shift to a deeper conceptual learning of math in college programs, PSTs are more apt to practice the same methods in their future classrooms (Li & Castro Superfine, 2018).

Future teachers need to learn to explain why math procedures work (Beckmann, 2014). They must be able to explain mathematical concepts in different ways and using different representations (Ambrose, 2004). In order to relay their own content knowledge to students, teachers must possess both common content knowledge (CCK), which is mathematical knowledge that most adults possess, and specialized content knowledge (SCK), which is the mathematical knowledge specifically necessary for teachers (Ball, 2003). In order to use these components of the mathematical knowledge of teaching in an effective manner, PSTs must understand the value of culturally responsive pedagogy. PST programs have the obligation to support future educators to be knowledgeable in their common and specialized knowledge of math content and to be capable of delivering instruction in a confident and culturally competent manner.

Research Question

My research question is:

1. What are the important factors to consider when developing instructional strategies that promote specialized content knowledge and the intrinsic motivation to teach for conceptual understanding among preservice elementary mathematics teachers?

Little is known about the nature of elementary teachers' mathematical knowledge of teaching (MKT), but most research studies focus on a single teacher or small samples of teachers. This study will also be limited to one class of PSTs, but it is an action research study. Action research is characterized by its purpose of generalizing knowledge to address the immediate needs of people in specific settings and also to improve practice (Herr & Anderson, 2015). The purpose of this study will be to improve my own practices in a classroom at a university.

It is the purpose of this study to examine ways to support my student PSTs in improving procedural and conceptual math content mastery, as well as the specialized content knowledge that they will need in order to feel empowered to teach their future students. The evidence collected through results of skills assessments in my classes has led me to choose multiplication and division of fractions as a focus for this study.

Teacher beliefs about the efficacy of conceptual methods of teaching mathematics versus procedural teaching methods need to be developed.

Purpose of the Literature Review

A literature review is a document that presents an argument that is logically organized and based on the current state of what is known about the research topic to be studied. It provides the context and background of all current knowledge of the topic (Machi & McEvoy, 2016) and summarizes and synthesizes all of the relevant ideas that are pertinent to an inquiry (Efron & Ravid, 2013). While I was attempting to establish the rationale for a study, I realized the need to shift the focus of my study because of the findings from the current literature.

I mainly used the ERIC and JSTOR databases as a source for literature as I began this research study on SCK of practicing teachers and PSTs. As I found and reviewed, articles, I saw that the reference sections for those articles provided more citations of relevant material to search. The university library provided sources for peer-reviewed journal articles and conference papers. I changed the focus of the study to include the MKT framework, as I discovered articles and conference papers by Deborah Ball. In addition to these sources, I found textbooks to include literature on culturally relevant teaching and gender issues in math. By widening the focus of the types of knowledge necessary for quality teaching of mathematics, I realized the necessity to narrow the focus to only PSTs. These sources helped me to develop the themes around which this study is focused and provided evidence of convincing arguments needed for a logical review and summary of the current knowledge of pertinent literature. This literature includes knowledge of the theories that provide the foundation for the study and studies to indicate the necessity of further research that will be done in this study.

Theoretical Perspective

The framework for this research study is based on the work of Lee Shulman and the mathematical knowledge for teaching framework developed by Hill, Ball, and Schilling (2008). Lee Shulman (1986) was the first to propose that the lack of student mathematical knowledge in the United States might stem from a lack of teacher pedagogical content knowledge, which is the ability to carry out the work of teaching mathematics. Hill, Ball, and Schilling (2008) introduced a framework for the different types of knowledge that are included in the constructs of MKT. The MKT is subdivided into subject content knowledge and pedagogical content knowledge, with each subdivision further divided into three narrower categories. The three categories of subject matter components are common content knowledge (CCK), specialized content knowledge (SCK), and knowledge at the math horizon. The pedagogical content knowledge components include knowledge of content and students (KCS), knowledge of curriculum, and knowledge of content and teaching (KCT). This research study will focus on the subject matter components of CCK and SCK. The framework is shown in Figure 1.1. This framework does not give information as to the type learning opportunities that help PSTs to develop this knowledge and therefore will be explored in this study.

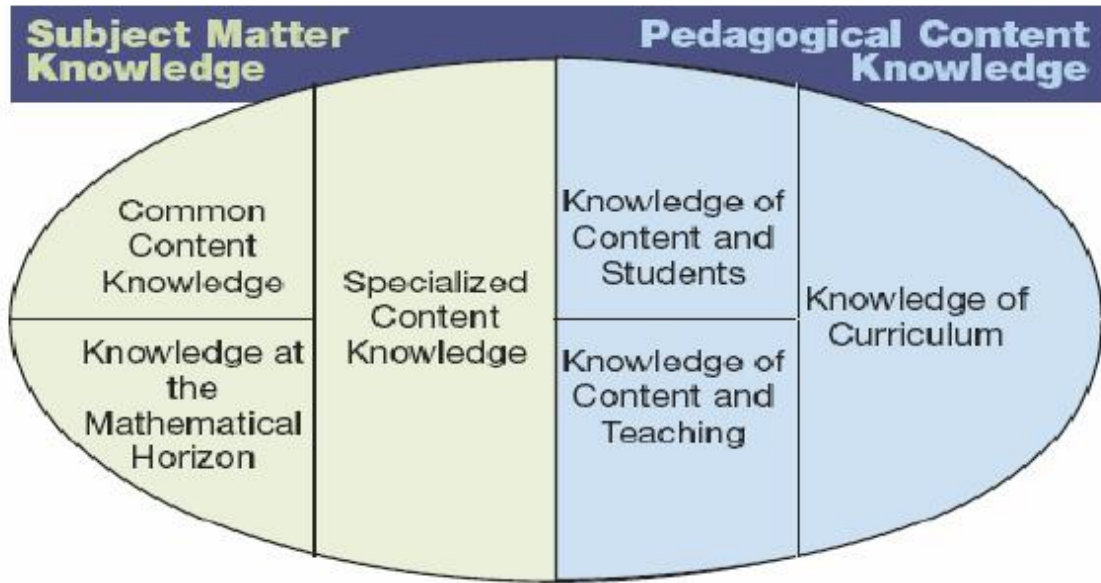


Figure 1.1 MKT framework for teaching.

For this study, I will use the definition of reformed pedagogy in mathematics that Smith (2013) defined as the change in teaching practice from the promotion of procedural understanding to the promotion of conceptual understanding. For decades the emphasis in mathematics classrooms was on procedural knowledge. This traditional method of teaching mathematics is based on rote memorization and the use of algorithms that simply mimic the teacher’s work (Shulman, 1986). This method provides no understanding as to why the algorithms are used and when they should be used in practical application. The methods of the reformed pedagogy seek to promote number sense and better understanding of mathematics concepts to better prepare future mathematicians (NCTM, 2000).

The reformed pedagogy movement continues through the support of teaching practices advocated by the National Council of Teachers of Mathematics (NCTM, 2009),

the Common Core State Standards Initiative (National Governors Association Center for Best Practices, & Council of Chief State School Officers. 2010), and the South Carolina College and Career Ready Standards (South Carolina State Department of Education, 2015) and includes an emphasis on student participation in meaningful problem solving, student collaboration, and multiple representations (Smith, 2013). This movement is also supported by the Association of Mathematics Teacher Educators (AMTE, 2016), which is devoted to the improvement of mathematics teacher education at all levels.

Cultural responsiveness is the last component of quality teaching that is considered in the study, and an approach that must be interwoven into the mathematics curriculum. Just as the reformed pedagogy gives students the ability to better understand mathematics as a tool to understand their own lives, culturally responsive pedagogy will give students a deeper understanding of their lives and the ability to see math as a tool to help make the world a more equitable place (Gutstein & Peterson, 2006). Teaching with cultural responsiveness gives all students the opportunity to apply the conceptual understanding of the reformed pedagogy that they have learned from a teacher with full MKT (Ladson-Billings, 2009). A more thorough explanation of each component is included in this chapter.

The PST Knowledge Gap

There is ample research that strongly indicates a gap in the knowledge that PSTs have in mathematics. Both knowledge and interviews suggest that elementary teachers in the United States vary in their understanding of the mathematics they teach (Hill, 2010). In a study of Chinese and American elementary teachers, Ma (1999) concluded that

Chinese teachers possess more MKT than American teachers. Ma continued the observation by saying that the knowledge of U.S. teachers is clearly fragmented. A study by the National Council of Teachers of Mathematics [NCTM] (2000) found that the big ideas of mathematical knowledge are beyond what most teachers experience in standard preservice courses.

Preservice courses for prospective elementary math teachers need to support PSTs in their own understanding of the deeper concepts of mathematics and not just the knowledge of algorithms for completing computations. Elementary teachers have procedural attachments, and they lack conceptual knowledge themselves in understanding fractions (da Ponte & Chapman, 2008). Teachers with this limited knowledge depend on textbooks for explanations of math concepts they do not understand (Sutton & Krueger, 2002, p. 15).

Actual assessment results of MKT of most PSTs have been done through research studies and not through any formal testing of prospective teachers. The Praxis II (Educational Testing Service [ETS], n.d.) is the instrument that most states use to test knowledge requirements of teachers, but the results of the tests are not public information. Assessment of the CCK and SCK of prospective teachers can be done within the teacher education classroom and can be used as formative or summative assessment.

Importance of PST Programs in Developing MKT

Math methods courses can increase the CCK and SCK of PSTs, as well as shape their beliefs about teaching math using reformed pedagogy (Ball, 1990). Much attention has been focused over the years on how to solve the problems of math deficiencies in

U.S. public schools. The focus has been on how to change the curriculum and standards, rather than on how to teach them (Ball, 1993). Teachers cannot teach what they have not had opportunities to learn. Designing courses in mathematical knowledge for teaching and implementing them is a huge task that must be done (Ball, 1993).

Most research supports the concept that PST programs will have the greatest impact on helping increase the MKT of teachers. This effort will require that math educators focus on identifying the MKT needed and then develop the courses necessary to support PST in developing high quality effective mathematics teaching skills (Ball, 1993). Teacher education programs must focus on where they will be most useful, and recognize the topics that most challenge PST, in order to affect positive change in future MKT (Hill, 2010). Cipra (1992) stated that it makes sense to attack the problems of elementary school teachers at the college level, since this is where all teachers expect to learn to teach.

Beyond simply knowing how to carry out basic math procedures, teachers need to be able to explain why math procedures work (Beckmann, 2014) and be able to provide examples of relevant application. Teachers of content courses for PST must help them to develop their own understanding. Experiences that accomplish this task are important for PST so that they learn the mathematical content and, at the same time, learn to use it as a model for their own teaching (Thanheiser, Browning, Moss, Watanabe, & Garza-Kling, 2010).

CCK

Common content knowledge (CCK) is the mathematical knowledge that most adults possess. One element of CCK is the ability to correctly recall and execute grade-level appropriate ideas and procedures (Hill et al., 2008). Rarely do mathematical commissions meet without noting that teachers require strong content knowledge in order to be effective in the classroom (Hill, 2010). Teachers who possess CCK are more likely to present material clearly and error-free (Ball, 1990). The first standard in the AMTE guide for well- prepared beginning teachers addresses the importance of CCK. Standard C.1: Knowledge of Mathematics for Teaching is

Well-prepared beginning teachers of mathematics possess appropriate mathematical knowledge of and skill in mathematics needed for teaching. They engage in appropriate mathematical practices and support their students in doing the same. They can read, analyze, and discuss curriculum, assessment, and standards documents as well as students' mathematical productions.

In a quantitative study by Heather Hill (2010), teachers were asked questions about the number 0 as one item to assess their CCK. The three questions were: Is 0 a number? (yes); Is 0 even? (yes); Can 8 be written as 008? (yes). These questions do not necessarily contain the common mathematical knowledge that most adults use, but they are a type of knowledge that is common across professions, such as accounting and engineering. One major finding in the Hill (2010) study was that CCK questions were easier for PSTs to answer successfully than were questions about SCK.

Another quantitative study on CCK and SCK was conducted on the PST knowledge of the calculus topic of derivatives. CCK was defined as a question that is answered without justification or using any representation (Pino-Fan, Godino, Font, and Castro, 2010). The results of their study showed that PST at this level had difficulties solving tasks not only pertaining to SCK but also with those pertaining to CCK (Pino-Fan et al., 2010).

SCK

From previous studies, scholars have concluded that teachers who are strong in math content can do more than simply solve problems for students. They can sensibly interpret and respond to student work products and can design and implement more conceptually grounded lessons (Fennema & Franke, 1992). Teachers who are stronger in math CCK are also stronger in SCK.

The criteria for what constitute quality SCK is varied according to researchers but lengthy in all cases. This is due to the amount of responsibility placed on a teacher to successfully design and implement a lesson for any given day. In her research study, Pettry (2016) included criteria such as representing content in a way that is accessible to all students and selecting activities that meet the needs of culturally diverse students as an important aspect for quality SCK. This criterion was in addition to the quality criteria listed in previous research by Hill (2010).

Hill (2010) stated that SCK focuses on job-embedded tasks such as responding to student work samples and selecting accurate representations and explanations. Hill (2010) also added additional criteria, including: knowing mathematical explanations for

common rules and procedures; constructing and/or linking nonsymbolic representations of mathematical subject matter; interpreting, understanding, and responding to nonstandard methods and solutions; using mathematical definitions or proofs in accurate yet also grade-level appropriate ways; and diagnosing errors in student work.

Although most of Hill's criteria for SCK were first developed by Hill, Ball, and Schilling (2008), their list of criteria also included recognizing student developmental sequences by identifying the problem types, topics, or mathematical activities that are easier or more difficult at particular ages, knowing what students learn "first," having a sense for what third graders might be able to do; and knowing common student computational strategies, such as using benchmark numbers or fact families.

Hill (2010) gave examples of questions for assessing SCK. Although most adults know algorithms to ascertain the correct answers, the question on this exam requires knowledge specific to the teaching profession. One item asks for the teacher to choose a diagram that does not represent the equation, $1\frac{1}{5} \times \frac{5}{6} = 1$.

Two more of Hill's questions in the 2010 quantitative study asked teachers to identify, based on student statements, the student who has the most advanced understanding of a given topic. The questions labeled SCK in this study were found to be significantly more challenging for the PST participants than the questions that were labeled as CCK.

Hill et al. (2008) found that a measure of MKT predicted student achievement. A study of first-year teachers in New York City (Rockoff, Jacob, Kane, & Staiger, 2008) concluded that SCK of teachers is a better predictor of student outcomes, even more so

than cognitive ability. Pedagogical content knowledge of teachers was found to be more predictive of student knowledge growth than was the CCK of the teacher (Baumert et al., 2010).

Reformed Pedagogy

In addition to measuring MKT of teachers, this study surveys the math knowledge of teachers and their prior learning experiences. Questions as to the beliefs PSTs hold about the traditional procedural methods of teaching and learning versus reformed pedagogy of using conceptual understanding will help me to gauge development of teacher attitudes and beliefs of the methods. In order to help PSTs develop MKT, math educators need to understand the currently held conceptions that their students have and to build on these conceptions to develop better mathematical thinkers (Ball, 1993). It is important that engagement in math is coupled with an intrinsic motivation to see the engagement as useful and worthwhile (NRC, 2001), And connected with a belief in one's own efficacy.

In teaching for conceptual understanding, educators must teach PSTs in the same way that they want them to teach their classes. Creating various representations for mixed numbers and fractions such as $1\frac{1}{5}$ or $\frac{2}{3}$ using a variety of manipulatives (such as Cuisenaire rods, base ten blocks, pattern blocks, fraction circles, etc.) leads to discussions of the unit whole (Thanheiser et al., 2010). This strategy is helpful in developing number sense and understanding of concepts that were previously missing from the MKT of PSTs.

Thanheiser et al. (2010) give an example of a problem that can be an assessment item for both CCK and SCK and can be found by using manipulatives or diagrams rather than equations or algorithms. For example:

A cookie jar is on the table. As each person comes by they take a part of the cookies remaining in the cookie jar. Al eats $\frac{1}{2}$ of the cookies. Bob later eats $\frac{1}{3}$ of the remaining cookies. Then Cal eats $\frac{1}{4}$ of what remains, and Don eats the last six cookies. How many cookies were in the jar originally?

Thanheiser et al. (2010) identified the CCK for this problem as understanding and solving problems with fractions, and they identified the SCK as understanding multiple representations of fractions.

In recent years, educators have made efforts to focus on the factors that produce future mathematically proficient students. The National Research Council (2001) set forth in its document *Adding It Up: Helping Children Learn Mathematics* a list of five strands essential to accomplishing this task. The five strands included conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The National Assessment of Educational Progress (NAEP) (2003) delineates what mathematical abilities are measured by its nationwide testing program. The abilities include conceptual understanding, procedural fluency, and problem solving.

Conceptual understanding is the first in the list of both documents. *Conceptual understanding* can be defined in many ways, but it is essentially the learning of mathematics with understanding and actively building new knowledge from experience and prior knowledge (Balka et al., 2003). Students demonstrate conceptual understanding

in mathematics when they provide evidence that they can generate examples of concepts, use models, diagrams, and a variety of other representations of concepts, and apply concepts, principles, and facts.

To help students gain conceptual understanding, teachers plan activities using hands-on materials, or *manipulatives*, to assist them in understanding abstract ideas. The teacher helps the students to use their prior knowledge to generate new knowledge and to use the new knowledge in unfamiliar situations (Bowens & Warren, 2016). Making students see connections between mathematical learning and prior learning is key to conceptual understanding. The idea of conceptual understanding is important in order to help students build foundational skills for higher math learning. Without it, there is no internalization of concepts and no ability to transfer information to new learning.

Conceptual understanding is built upon discovery: learner-centered activities where the teacher facilitates and takes a less active role (Balka et al., 2003). These ideas are certainly not new to education, but they are based on ideas of progressive curriculum theorists from the early 1900s. The roots go back to theorists, such as John Dewey, whose ideas have dominated American schools since the early 20th century. He believed that children should learn by doing (Schiro, 2013) and that they should make meaning and construct knowledge as results of their experiences. Since children are to be actively engaged in their own learning, he proposed that teachers' jobs should be to prepare educational environments that engage children to learn and construct meaning (Schiro, 2013).

The impact of constructivism can be seen through the support of professional education groups such as the National Council of Teachers of Mathematics (2000) and the National Research Council (2001) who promote the learner-centered ideology. The National Association for the Education of Young Children (NAEYC) published the guidelines for several developmentally appropriate practices to bring about this ideology, which included hands-on learning, use of concrete manipulatives, learning centers, thematic units, integrated curriculum, cooperative learning, mixed age grouping, partnerships with friends and communities, multiple intelligence recognition, and culturally responsive teaching (Novick,1996).

Another important progressive who took a stance for educational reform was Maria Montessori, who established nontraditional schools that centered around the development of children. She developed a curriculum that utilized hands-on manipulation of materials. Montessori believed that children learn through their environment, which leads to sensory understanding (Lillard, 2011). Piaget (1973) also touted the importance of engaging in creative, inventive work and declared it crucial for developing deep understanding.

In the 1980s, the NCTM reinforced the themes of the 1920s progressive education and advocated student-centered, discovery learning. The “variant” of progressivism favored by the NCTM at this time was called constructivism, and it is this learning theory, under the broad curriculum theory of progressivism, that frames this study. Constructivism as it applies to education means that knowledge is gained by self-discovered knowledge, which is reliably understood and remembered (Klein, 2003).

Other leaders in mathematical education supported constructivism, such as Piaget, with his ideas of developmental learning, and Vygotsky (1987), with his “Zone of Proximal Development,” which is concerned with child-centered, cooperative learning. When the National Council of Teachers of Mathematics distributed the *NCTM Standards*, they were supportive of constructivist pedagogy. The NCTM defined the role of the teacher as one in which the teacher initiates activities on which children reflect and abstract patterns of regularities for themselves. The constructivist theories did not go without challenge, as there were always those critics who pointed to failures of children to learn basic skills.

Ball (1996) wrote that professional curriculum developers advocate a constructivist approach for students, yet they only gradually allow teachers to learn in a constructivist fashion. I chose to frame this research study with the progressive curriculum perspectives that are concerned with the constructivist learning ideologies. Research is plentiful to support the need for reform in PST education in order to help teachers understand the constructivist ideas and the efficacy of conceptual understanding teaching strategies. There is also research by those who oppose the theories and criticize the results of the implementation of these ideas in public education.

Culturally Responsive Pedagogy

Gloria Ladson-Billings (2009) advocated that educators recognize and address the cultural differences between teachers and students. She defined this practice as “culturally responsive pedagogy.” This is an approach to teaching that advocates for teachers to consider the cultures of their students in planning lessons and in determining

the methods by which they are taught (Emdin, 2016). Connecting content to context is a valuable skill for teachers to use to engage students in math lessons. Christopher Emdin (2016) stated that when students become fully engaged with the lesson, curiosity is awakened, and they begin to ask questions with deeper connections that go beyond the scope of the traditional lesson. Emdin (2016) described an example of a colleague who used pictures of an elevator in a housing project to help students better understand geometry concepts.

Conventional pedagogy has often been considered by many students as a subject completely disconnected from their daily lives and from any efforts that they apply to make sense of the world (Tate, 1994). Teachers need to be able to use an extensive range of expertise in all of the theories in the theoretical framework in a manner that supports the learning of all students in their diverse classroom populations. By drawing on the cultural practices and prior knowledge of the youth in their classrooms, teachers must support students in being able to engage in meaningful academic discourse (Nasir, Rosebery, Warren, & Lee, 2006).

Recognizing the importance of practicing pedagogy that empowers every student is an interconnected component of cultural responsiveness and reformed pedagogy. It is particularly important to minority students and/or students living in poverty because they are overrepresented in classrooms that emphasize worksheets, rote memorization, and computer skills- based instruction, rather than problem-solving tasks and lessons (Oakes, 2008). According to AMTE (2016), “Programs that focus on the mathematical content knowledge of beginning teachers of mathematics are directly addressing the issue of equity”(p. 38).

Historical Perspective

The math education policies and programs for United States public schools has been a point of contention for decades. The struggles have been mostly between content and pedagogy. *Content* is what is taught, and *pedagogy* is how to teach it. It seems that the two would obviously work together in curriculum decisions, but the source of conflict is which decision to make first (Klein, 2003).

The roots of the conflict go back for years, as the constructivist ideology of progressive educators was embraced and then dismissed, all based on the events of the time period, politics, and the public opinion swayed by these factors. Although ideas of progressive education have been a part of American education since the early 1900s, there were always those who challenged its merits. The challenges increased during times of national crisis. Wars and other matters of national security and dominance as a world power influenced politicians and the public to scrutinize the public education system and to call for changes in the curriculum of the time (Raimi, 2000). In the 1940s, it became a public scandal that army recruits had to be trained in arithmetic skills that were needed for basic duties of military personnel (Raimi, 2000). In the 1950s, the Union of Soviet Socialist Republic launched Sputnik, the first space satellite, and the United States became not only alarmed by the idea of losing its dominance in the space race but also embarrassed at losing the race to be first to accomplish the feat (Klein, 2003). Ongoing disagreement between people supporting “back to basics” versus “progressive education” movements continued through the latter part of the 1900s. In 1989, the NCTM promoted their standards and represented their view of what American children should learn in mathematics classes. The standards reinforced the progressive themes of student-

centered, discovery learning. Both basic skills and general math principles were to be learned through “real world” problems (Bosse, 1995). This stance was a compromise to people on both sides of the issue of math education practices and policies.

In 1989, President George H. W. Bush made a commitment at an educational summit to make U.S. students first in the world in math and science by the year 2000. The National Science Foundation proposed that the NCTM standards be the blueprint for change in math and that the standards were the key to implementation of changes (Raimi, 2000).

The 1990s became a contentious time for math educators, as there was extended disagreement between proponents of basic skills versus those who favored conceptual understanding of mathematics. Hung-His Wu (1999), who explained the essential connection between the two in his article “Basic Skills Versus Conceptual Understanding: A Bogus Dichotomy in Mathematics Education,” called the separation of the two types of mathematical learning was called misguided. Not until the implementation of Common Core Standards did ideas of student-centered learning, discovery lessons, and conceptual understanding regain national attention as the primary focus on public school education.

Other debate has centered around the type knowledge that is most important for teachers to possess, content knowledge or pedagogical knowledge. Lee Shulman (1986) was the first to argue for the importance of pedagogical content knowledge and the first to recognize that this was a missing paradigm from teacher education programs. Yet even today, not everyone has bought into the necessity of the importance of knowing how to

teach. Some of those who believe that CCK is the most important factor for teachers may point to the success of alternative certificate programs or programs such as Teach for America. These programs use college graduates with many different types of degrees and put them in classrooms, where they learn sometimes little to no pedagogical content. Often these hires become successful teachers, and often they leave the classroom after a short time (Strauss, 2013).

In 1999, supporters of both “back to basics” mathematics and conceptual mathematics pedagogy embraced Liping Ma’s book, *Knowing and Teaching Elementary Mathematics*. In her book, she explained the interrelationship of pedagogy and content at the elementary level (Ma, 1999). Since that time, the importance of pedagogical knowledge has been the focus of mathematics education research due to the theoretical framework of MKT (Hill et al., 2008).

Teaching Effectively for Diverse Populations

Issues of equitable teaching arise in the methods that teachers use to teach children and in the distribution of high-quality teachers in high poverty areas. The problems that arise in teaching children from diverse groups is that many students do not see the relevance of traditional math programs and therefore are not motivated to learn the content. Children who are labeled as below basic in mathematics on standardized testing are often relegated to remedial math classes. Repetitive drill and endless worksheets are the methods chosen to increase the achievement levels of these children by teaching them the basic skills (Delpit, 2012).

Lisa Delpit (2012) explained in her book, *Multiplication is for White People*, that the children of high poverty families are often adept at problem solving due to their family situations. Young children in these homes often fix their own meals, clean up their own spills, and take care of younger siblings. Rather than meeting these children where they are and using their prior experiences and strengths, these children are set up for failure in a math classroom where the learning becomes a disconnect for them. These are the children who are often placed in remedial math classes to increase their basic skills knowledge. The teaching strategy to accomplish this task is often the use of tedious worksheets and computer programs that are not connected to other skills, not relevant to the student's prior learning or life experiences, and do not motivate or inspire students to want to learn more. At the same time, other children are in classrooms performing rich problem-solving activities. The end result is a widening of the achievement gaps between the two groups.

Conceptually based foundational curriculums can encourage students “to critique answers, question assumptions, and justify reasoning” (Gutstein & Peterson, 2006, p. 5). The traditional rote calculations, drill and practice, and worksheets eventually lead to a disconnect for students who find them irrelevant and not connected to their social reality. These traditional drill assignments have no context and only teach students to perform a particular procedure over and over, with no idea of when and how to use the skill. This robs students of tools to help them participate in society (Gutstein & Peterson, 2006).

Children in higher poverty districts are often in the lowest performing schools because they do not have access to high quality teachers. With a nationwide teacher shortage looming, the high poverty, predominantly minority schools will be most

affected. Quality principals are a major factor in recruiting and retaining quality teachers, but the turnover rate for principals in these areas is at a high, with an annual turnover rate of 15–30% (Pendola & Fuller, 2017). Other factors that affect the inequitable distribution of higher quality teachers are poor working conditions and lower salaries than in more affluent districts.

My own state of South Carolina is the home of the “Corridor of Shame,” where schools along the I-95 corridor were found to be severely neglected because of inequitable state funding. There was a never-ending struggle against the poor conditions, turnover in teachers, and poor student achievement, all due to a lack of a tax base to support property taxes. South Carolina based funding on collected property taxes, which was negligible in rural towns with little industry (Findlay, 2017). Universities can help to alleviate the problem of the availability of quality teachers by strengthening their teacher education programs to include criteria for CCK, SCK, reformed pedagogical knowledge, and knowledge of cultural differences.

In discussing the cultural differences among populations in South Carolina schools, I have been referring to differences in racial, ethnic, religious, or socioeconomic status of students. Gender differences should not be ignored. Girls often have poor concepts on their own abilities in math and therefore are often reluctant to continue to study math at more rigorous levels when they get to high school (Bell & Norwood, 2007). Richardson and Suinn (1972) defined math anxiety as “involving feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of life and academic situations,” a phenomenon experienced most often by female students. Elizabeth Fennema (2000) summarized what

she knows about gender differences in learning mathematics by describing the inequitable teaching methods for males and females. She further explained that boys are more likely to use abstract strategies, even in early grades, while girls tend to need more concrete strategies, such as modeling and counting.

Research has found that the keys to success for girls in mathematics lie within confidence and self-esteem. Clewell, Anderson, and Thorpe (1992) believed that females are more influenced by what they believe their teachers think of them than are their male counterparts. Elementary PSTs who have a low level of confidence in math often spend less time in teaching math and more time teaching in a traditional abstract manner. These teachers thereby perpetuate the cycle of producing students with little conceptual mathematics knowledge and the inability to progress appropriately in further math learning. Building confidence in the self-efficacy of elementary PSTs is a critical piece in producing quality teachers from teacher education programs.

Research Methodology

A qualitative study conducted by Pettry (2016) to examine the development of SCK among secondary mathematics PST determined that the development of SCK is strongly influenced by interactions with learners. The 47 participants completed an SCK assessment consisting of 22 questions relating to algebra and a prospective teacher survey consisting of questions on demographic and background information. Face-to-face interviews were conducted as a follow-up to the assessments in order to gain clarity into teacher responses and to delve more deeply into the field experiences of the participants

(Pettry, 2016). Similar research has been done to determine the effects of and on SCK at different levels of mathematics study.

In a study of PSTs competencies on fraction multiplication, Son & Lee (2015) attempted to understand the knowledge in three different contexts (1) a word problem format, (2) a purely symbolic notation format, and (3) a format requiring the use of visual representation. The study revealed distinct differences in the levels of competency of the PSTs, ranging from PSTs who had no correct responses to those who were able to portray multiplication in all three aspects (Son & Lee, 2015)

A study by Depaepe, Torbeyns, Vermeersch, Janssens, Janssen, Verschaffel, & Van Dooren (2015) examined the content knowledge and pedagogical knowledge of both elementary and secondary teachers focusing on rational numbers. The results showed a significant difference in the superior performance of secondary teachers on content knowledge but showed little difference between the two in their pedagogical content knowledge.

Research involving PSTs' knowledge of fractions is important because it is typically a problematic area of study for both elementary teachers and their students (Ma, 1999). Although there has been some research to suggest the need for additional support for PSTs to acquire the CCK they need for effective teaching practices, there is little research on how to improve the CCK of PSTs (Olanoff, Lo, & Tobias, 2014). My research differs from others in the intervention three-stage approach of the preservice education class study. During the intervention for this study, the participants had opportunities to serve as learners, reflective practitioners, and teachers. This study was a

qualitative case study used for action research. Action research is research done by researchers or in collaboration with practitioners or community members (Herr & Anderson, 2015) to improve their own practice. The research process followed is collaborative, reflective, and cyclical. This study focuses on improving the MKT for PSTs and is a classic example of research done in the practitioner's setting, in collaboration with colleagues to affect change in teaching practices of the researcher.

I chose a qualitative case study as the best option for this research because of the alignment of the purpose, data collection tools, data analysis methods, and intervention plan with that of the characteristics of quality criteria for the particular research design. The data collection tools of a teacher reflection journal, daily student feedback from exit slips, and meaningful conversations and debriefing sessions with participants and colleagues provided multiple opportunities for reflection. The plan development, implementation, observation, and reflection create a cycle of activity that continued throughout the research process.

Case studies are of a qualitative design and develop an in-depth study of a case, program, event, activity, process, or one or more individuals (Creswell, 2014). This research is an in-depth study of a PST class at a small university, with a class of 30 second-year college students. Methods of data collection are consistent with those of qualitative studies. Included in the data collection tools are surveys, observations, videos, and lesson plans. I used observations and formative assessments throughout the course of the intervention to measure growth in participant development of MKT and I reported the results using a rich description of the data findings.

I completed data analysis through a priori coding of data, which is a common process in qualitative research (Creswell, 2014; Saldana, 2009). To complete the coding for this study, I used the evidence gathered from a teacher-made rubric of criteria for high quality teaching using components of CCK and SCK. I selected criteria for the rubric based on the theoretical framework of MKT and explanations of its components by Hill, Ball, and Schilling (2008). More details of the data collection process and the data analysis are provided in Chapter 3.

Summary

The literature review provides evidence of a need for PST courses to increase MKT because of the potential positive impact it has been found to have on student achievement. Hill et al. (2008) distinguished between two different types of knowledge—subject content and pedagogical content—and further delineated each type into three more categories. Evidence from the literature review shows that CCK is essential for teachers to understand in order to teach children, and SCK is vital for teachers to possess in order to increase student learning. The literature showed that teacher SCK has a greater impact on student achievement than does CCK but that those teachers with strong SCK also possess strong CCK (Rockoff et al., 2008).

Evidence from the review shows that there has been an ongoing battle between proponents of traditional procedural teaching methods in math and those of reformed methods of teaching for conceptual understanding (Raimi, 2000). There is evidence in the literature of the benefits of conceptual teaching methods in math as far as understanding, retention, and transference of concepts and ideas (Crocco & Libresco, 2007). The review

does not offer suggestions that procedural fluency of traditionally memorized basic facts of arithmetic is not important and should not be part of the curriculum. The implication of the review is that conceptual understanding of math concepts is critical for new learning, so that the more abstract concepts may be grasped as the student is sufficiently prepared to learn them.

The idea of cultural responsiveness is a factor in developing SCK, in that teachers can discern what questions, activities, and assessments are most appropriate for those with math anxiety issues and with a culture that differs from the teacher's. Choosing quality instructional materials and activities is one of the criteria for SCK presented by Hill, Ball, and Schilling (2008).

The literature is mostly concerned with the types of knowledge, what each type means, and the importance of each type to the teaching profession. There is not an abundance of literature that evaluates the strategies of how these areas of knowledge (CCK and SCK) are most effectively developed in PSTs and how teacher education programs can best support future teachers. The issues of reformed pedagogy and culturally responsive teaching have not previously been so closely tied to MKT as interwoven pieces of the theoretical framework. The focus of this research study is the idea of supporting teachers in preservice programs to increase knowledge of SCK that will best serve all children.

The implications of the review findings led to the planning and implementation of a research design to study the problem of practiced and will be discussed in Chapter 3.

Chapter 3

Research Design and Methods

Quality teaching of mathematics depends on the teachers' knowledge of the subject matter and the teachers' abilities and knowledge to effectively teach mathematics to students (Ball, 2003). Research from my literature review and my own experiences as a math teacher have indicated to me that undergraduate PSTs are not provided with a solid background in conceptual mathematics. This is a common occurrence that can lead to significant challenges for these future teachers as they learn to teach mathematics. It is the purpose of this study to examine ways to support my student PSTs in improving procedural and conceptual math content mastery, as well as the specialized content knowledge that they will need in order to feel empowered to teach their future students. The evidence collected through results of skills assessments in my classes has led me to choose multiplication and division of fractions as a focus for this study. Teacher beliefs about the efficacy of conceptual methods of teaching mathematics versus procedural teaching methods need to be developed.

Mathematical Content Knowledge for Teaching (Hill et al., 1988) components Common Content Knowledge (CCK) (Hill et al., 1988) and Specialized Content Knowledge (SCK) (Hill et al., 1988), Reformed Pedagogical Practices (Smith, 2013), and Culturally Responsive Teaching Practices (Ladson-Billings, 2009) are the theories that form the theoretical framework of this research study. Within the reformed pedagogical

practices are embedded theories on best practices, conceptual understanding, and social justice.

I used the practices and elements of the primary theories with their embedded components in a three-phase intervention that led to a better understanding of the research questions related to this study.

The research question for my study is

1. What are the important factors to consider when developing instructional strategies that promote specialized content knowledge and the intrinsic motivation to teach for conceptual understanding among preservice elementary mathematics teachers?

Through a qualitative case study design that included a three-phase intervention plan, I addressed these questions. Phase 1 was the experiencing pedagogy phase, where the PSTs were learners and the primary function of the phase was to increase the CCK of the PSTs. Phase 2 was the reflecting on pedagogy phase, in which students were beginning to develop SCK and beginning to make meaning of their own learning. Phase 3 of the intervention was the practicing pedagogy phase, where PSTs participated as practitioners planning and implementing a lesson. In this phase, PSTs gave evidence of their SCK gained during the intervention, and an analysis of their work led to the development of an inductive rubric to assess the classwork and to form instruction for my future work as a practitioner.

This chapter describes the components of qualitative action research necessary to conduct a thorough study of the research questions. I included a descriptive rationale of

the qualitative research design, along with a detailed description of the intervention plan. Information on the participants and a justification for the sample selection follow the information on the intervention plan. I first described the characteristics of the whole participant group; then I provided more detailed information for a smaller group of students, whom I selected for the study focus due to results gathered during the research study process. In this chapter, I explained data collection methods from multiple sources consistent with qualitative research and provided an analysis plan of the raw data. In addition, I provided the plan for coding the data using several cycles and then weaving the themes into a rich description that tells the story of the action research, which makes connections of all components to the problem of practice, the theoretical framework, and the research questions.

Participants

Participants in the study are PSTs in one class at a small university in South Carolina. The university requires students who are seeking a degree in elementary education to successfully complete a three-course sequence in math content knowledge. The first course is focused on place value, whole number meaning, and whole number operations; the second is focused on rational number meanings and operations; and the third course focuses on geometry and measurement. Operations on fractions have historically been the greatest area of weakness for PSTs enrolled in this course sequence. This realization provides the motivation for the problem of practice and research questions, and a rationale for using these students as participants for this research study.

The students in this study were in the second course of the sequence and therefore have prior knowledge of the meanings of addition, subtraction, multiplication, and division. The sample is a purposeful sample since I chose the students for the study based on a specific purpose. The students chosen for the sample were predominantly female and majority White. The racial makeup of the sample was approximately 80% Caucasian and 20% African American. Although the glass makeup is not representative of the diversity of the university, it is unfortunately representative of the population of teachers in the United States. The racial and ethnic demographics of the student population in today's mathematics classrooms is significantly different from that of their teachers, who are majority female, white, and monolingual (U. S. Department of Education, 2013). Most students are from in state, although there were students from other states and countries in the class. Most out-of-state and international students were at the school on athletic scholarships. Students in the study volunteered to participate in the research, with the option to withdraw at any point in the study.

Based on the results of the intervention data, I chose to focus much of the research analysis on nine students. These students were a purposefully chosen sample based on their performance during and after the intervention. Each presentation group consisted of three students. I chose the three students from the high-performance group, which showed the most SCK from the intervention plan. I also selected a middle-performing group and the lowest performing group of students. I focused on the data collected from these nine students to attempt to ascertain what factors made some students more successful in gaining SCK than other students in the class.

Individual Student Profiles

The names of each student described in the study focus are all pseudonyms to protect the anonymity of the participants.

Ann. Ann is a 24-year-old White female from a small town in South Carolina. She is a single mother with an eight-month-old baby. She frequently brings the baby with her to see me during office hours or when she has make-up work to do after the class time. She brings a blanket and toys for the baby, who plays on the floor while her mother works. In class, Ann sits on a row with two other students who are conscientious and very serious about their coursework. Collaboration is encouraged for most class time, and her small group works well to complete tasks. Ann is a junior elementary education major.

Maggie. Maggie is a junior elementary education major from a small upstate town in South Carolina. She is 21 years old and a White female. Her work is always above average, and she is willing to share her opinions and ideas in class. During the class time, she chooses to sit with one other student with whom she collaborates, but the two are welcoming to any students who ask to join them in class activities.

Kate. Kate is a 19-year-old White female from South Carolina. She is a sophomore majoring in early childhood education. In class, she does not display characteristics of maturity for a college student. She is often giggling or on her laptop, even at times when a laptop is not part of the necessary materials for the class activity. I frequently walk towards her area of the room to keep her focused on the task at hand. Her class partner is not seemingly bothered by these behaviors, but neither is she benefitting from her collaboration with Kate.

Reagan. Reagan is repeating this math education course, since she was not successful in earning the requisite C- in her first attempt. She is a White female, who is a 22-year-old early childhood major. Reagan is very quiet in class and never offers her opinion or ideas to the full group. She sits with one other person in class who is younger than she is and who is very confident in her own abilities. Reagan seems to be a little intimidated by any students with whom she is placed in a group.

Cat. Cat is a younger student than most of the students in the class. She is a 19-year-old, White junior majoring in early childhood education. Her home is from a small town in South Carolina. Cat is in the same collaborative class group as Ann. She shows maturity and a genuine interest in her chosen major studies.

Alison. Alison is the athlete in the group. She is a softball player on the university team and has good support from coaches to perform well in academics. Alison is a 21-year-old senior majoring in early childhood education. In class, she chooses to sit alone at the back of the room and focus on the class work. When she is asked to work with a partner or a group, she is very willing to move around and participate with others. Her strategies for performing well in class may come as a result of advising from her athletic tutors and coaches.

Molly. Molly is an outspoken 19-year-old, White female majoring in elementary education. When she does not immediately grasp a concept, she becomes frustrated and angry. She is from a larger town in South Carolina and is an only child.

Amy. Amy is a White female from the university town. She lives with her parents and commutes to school. She is 20 years old and is a sophomore special education major.

She is quiet in class and never volunteers her ideas or opinions in whole class discussions. During class, she collaborates with two other students, neither of whom have strong math skills. One of the students is always the group spokesperson. When Amy is called upon to share, her response is always that she agrees with her spokesperson for the reasons she gave.

Joy. Joy is a 19 year old from a small town in the central part of South Carolina. She is a White, female, elementary education major who always gives the impression that she has more important things to do than attend class. She frequently offers her opinion and has often made others unhappy with her tactless criticism of their opinions.

Teacher/ Researcher Dual Positionality

The dual positionality of being the teacher as well as the researcher is a double-edged sword in that there is no one better to research how to improve a teacher's practice than the teacher. The positive benefit of the dual positionality is that it is an investigation that warrants deep and thorough reflection into the teacher's own practices. The downside of the positionality is that the participants often will respond in ways that they feel the teacher/researcher wants to hear. In some data collection, it has been necessary for me to insist that students remain anonymous when giving their written comments to ensure that the students give their honest opinions. The weakness in using anonymous opinions at all stages is the inability of the researcher to compare and analyze individual progress and to find meaning in the results. I gathered most of the data through clearly identified responses, and used member checking to clarify student intentions or to gain better insight into data.

Colleagues supported me in the data collection and analysis process by helping me to reflect on my daily lessons and by helping me see patterns or themes that were evident in the data during the coding process. One colleague from the English faculty helped to ensure that the written descriptions were related to the problem of practice, the theoretical framework, and the research questions. Having input from a math colleague was invaluable in my reflections on the effectiveness of the lessons and activities chosen for the interventions. The spirit of partnership is a characteristic of quality criteria for action research (Herr & Anderson, 2015) that is in evidence through the collegial partnerships and member checking that were a part of this research study.

Research Design & Methodology

Efron and Ravid (2013) defined action research design in education as an inquiry done by an educator in their own setting to advance their own practice. Teachers take on the role of researchers and study their own practices within their classrooms, programs, or schools when conducting action research. It is different from traditional educational research in that it is constructivist, situational, practical, systematic, and cyclical (Efron & Ravid, 2013). The characteristics of this research study are consistent with those of action research, since I am generating my own knowledge to understand the unique perspective of my class and students, and I am generating questions based on my own concerns in an intentional and systematic way using the cyclical steps of plan -act-observe-reflect.

The approach to action research used for this study is a qualitative case study. This methodology is described as a strategy of inquiry in which the researcher attempts an in-depth exploration of a program, event, activity, process, or one or more individuals

(Efron & Ravid, 2013). In this qualitative case study, the phenomenon to be explored is a PST class in a university setting. The problem of practice is addressed by quality action research, which seeks to find solutions for authentic problems and to empower people concerned to acquire relevant knowledge to be shared with others (Stern, 2014).

The research questions are situational, practical, and cyclical, all of which are characteristic of action research studies (Efron & Ravid, 2013). In a qualitative case study research design, the most appropriate type of research questions are of “how and why” forms (Yin, 1994). My research question is:

1. What are the important factors to consider when developing instructional strategies that promote specialized content knowledge and the intrinsic motivation to teach for conceptual understanding among preservice elementary mathematics teachers?

Qualitative researchers tend to collect data at the site where the participants experience the problem (Creswell, 2014). Case studies are of a qualitative design and develop an in-depth study of a case, program, event, activity, process, or one or more individuals (Creswell, 2014). This research is an in-depth study of a PST class at a small university, with a class of 27 second-year college students seeking a degree in elementary education. Methods of data collection are consistent with those of qualitative studies. Included in the data collection tools are surveys, observations, and lesson plans. I used observations and formative assessments throughout the course of the intervention to measure growth in participant development of MKT, and I reported the results using a rich description of the data findings.

I chose a qualitative case study as the best option for this research because of the alignment of the purpose, data collection tools, data analysis methods, and intervention plan with that of the characteristics of quality criteria for the particular research design. The data collection tools of a teacher reflection journal, daily student feedback from exit slips, and meaningful conversations and debriefing sessions with participants and colleagues provided multiple opportunities for reflection. The plan development, implementation, observation, and reflection create a cycle of activity that continued throughout the research process.

I completed data analysis through a priori coding of data, which is a common process in qualitative research (Creswell, 2014; Saldana, 2009). To complete the coding for this study, I used the evidence gathered from a teacher-made rubric of criteria for high quality teaching using components of CCK and SCK. I selected criteria for the rubric based on the theoretical framework of MKT and explanations of its components by Hill, Ball, and Schilling (2008). From my past teaching experiences, I have concluded that this setting is appropriate for this research study and provides optimal opportunities to address the research questions. Previous qualitative researchers have studied the development of SCK among different levels of students using various interventions and data collection processes.

A qualitative study conducted by Pettry (2016) to examine the development of SCK among secondary mathematics PST determined that the development of SCK is strongly influenced by interactions with learners. The 47 participants completed an SCK assessment consisting of 22 questions relating to algebra and a prospective teacher survey consisting of questions on demographic and background information. Face-to-face

interviews were conducted as a follow-up to the assessments in order to gain clarity into teacher responses and to delve more deeply into the field experiences of the participants (Pettry, 2016). Similar research has been done to determine the effects of and on SCK at different levels of mathematics study.

My research differs from others in the intervention three-stage approach of the preservice education class study. During the intervention for this study, the participants had opportunities to serve as learners, reflective practitioners, and teachers. This study was a qualitative case study used for action research. Action research is research done by researchers or in collaboration with practitioners or community members (Herr & Anderson, 2015) to improve their own practice. The research process followed is collaborative, reflective, and cyclical. This study focuses on improving the MKT for PSTs and is a classic example of research done in the practitioner's setting, in collaboration with colleagues to affect change in teaching practices of the researcher.

Intervention

Two weeks prior to the intervention, a test was given to PSTs to assess their prior knowledge of CCK and SCK. The questions for the CCK section were teacher made and based on released skills check assessment items given by the university in past years and from the released items of University of Michigan MKT measures. I based the criteria on the University of Michigan test, which was determined from studies by Deborah Loewenberg Ball, Mark Hoover Thames, and Geoffrey Phelps (1980).

The intervention was a series of 10 lessons over a period of five weeks. Before the intervention began, I administered a pretest to determine benchmark CCK of preservice

students on fraction operations, along with an attitudes and beliefs survey as to their own mathematical confidence levels. The first phase of the intervention was called the *learner* phase, in which students were to *experience pedagogy as students*. During this phase, I assumed the role of teacher, and the PSTs became the learners, who were engaged as active participants in the learning process to increase their conceptual understanding of fractions.

The structure of the intervention for the first three lessons was for me to engage students in authentic math tasks focused on fractions and students as learners. During the first three lessons, I modeled the criteria for SCK in lessons that actively engaged the participants in conceptual mathematics activities to increase their CCK. To help increase the CCK of fractions for PSTs, pattern blocks, cuisenaire rods, and fraction circles were all used to represent fractions in multiple ways. The lessons were chosen to increase the PSTs' conceptual understanding of fractions that were parts of different size wholes and different shaped wholes. I also devised lessons for PSTs to see a visual representation of obtaining equivalent fractions, rather than focusing on an abstract procedure of finding them. Although this phase was designed to focus on CCK, it was difficult not to bring in some discussion of pedagogy during these lessons. My goal as the teacher was to model the methods that the PSTs would be expected to use in the future and to present the methods in an effective manner that would presently help my PSTs to increase their own CCK. Table 3.1 shows the daily lesson plan for this phase.

Table 3.1

Daily Lesson Plan for Phase 1

Lesson Number	Objectives TLWBAT	Instructional Strategies	Structure	Assessment
Lesson 1	Represent fractions with different meanings and with multiple types of manipulatives.	Discussion of fractions as missing factors, parts of a whole, ratio, and probability.	Partners work together and share work with other pairs; whole group sharing.	Observation
	Evaluate the effectiveness of concrete materials used to represent fractions	Creating multiple representations of $\frac{2}{3}$ with fraction circles, pattern blocks, Cuisenaire rods, two color chips, and number lines.	Small Group	Whole group discussion
		IMAP video of elementary students using fraction concepts		Observation
		Closure		Journal reflections written at the end of class on the value of using each of the types of manipulatives.
Lesson 2	Represent equivalent fractions using concrete materials	Van de Walle (2007) fair sharing activity	Partners work together on each activity	Reflection on each activity shared with a group of 4 and shared with the whole class.
		Billstein (2016) activities using different concrete materials to discover ways to show equivalence of fractions		Teacher observation
		NCTM Illuminations activity		
		Closure – write reflections comparing the activities using concrete materials and conceptual understanding to traditional methods of rote memorization and algorithms from your perspective as a learner.	Individuals	Reflections in journal

Lesson 3	Find equivalent rational numbers using fraction strips and using reasoning skills.	Desmos online activity called Polygraph Rational Numbers.	Whole group.	Student responses (anonymous to class) available to whole class through the Desmos activity
		Closure—Reflections on individual beliefs about concrete math instruction and personal learning style.		Journal reflections

Participants used exit slips to answer questions about the efficacy of the activities and their understanding of the lesson concepts. I kept a reflection journal to record observations and feelings about each daily lesson in order to reflect on what learning had taken place and what would be the next steps in the process.

The second phase of the intervention focused on making sense of pedagogy. The PSTs transitioned from experiencing pedagogy to *reflecting on pedagogy*. I transitioned from the role of teacher to facilitator. For the first two lessons of this phase, students used the models of quality lessons that they had observed and the CCK that they had gained from Phase 1 to define quality criteria for effective teaching practices and then evaluate lessons of others based on the criteria. On the first day, the task was accomplished through small collaborative groups who brainstormed criteria and then agreed upon a final list to post on chart paper for a gallery walk. The groups viewed the list of other groups, made notes on findings, and commented on the work of others with Post-It notes. Through a whole group discussion, the class reached consensus on an extensive list of criteria that could be used for evaluation purposes. Among the listings were criteria compatible with culturally responsive teaching, such as making connections to a student’s prior experiences, engaging students in learning, and creating relevant lessons for student groups. PSTs used

the next day of the intervention as an opportunity to evaluate the lessons of others from selected videos using the evaluation checklist that they had developed. Breaks in the lesson provided time for reflection on the lesson components and on the value of the reformed pedagogy based on conceptual understanding.

Students completed the first step in small group discussions and the second step in a whole group setting. The whole class reached a consensus to create an evaluation sheet for observations of math lessons. As the facilitator of the discussions, I had opportunities to suggest ideas of high-quality criteria that Deborah Ball (2003) outlined in her mathematical tasks for teaching. Ball (2003) included designing accurate and useful explanations, representing ideas with physical models, graphical models, and/or symbolic notation, and posing good mathematical questions and problems that are productive for students' learning. The daily lesson plan for day 1 of Phase 2 is found in Table 3.2

Table 3.2

Daily Lesson Plan for Day One of Phase 2

Lesson number	Objectives TLWBAT	Instructional strategies	Structure	Assessment
Lesson 4	Identify quality criteria for effective teaching in math lessons	List quality criteria and post ideas on chart paper	Small group	Observation
		Walk around to read ideas of each group, comment to each group, and make notes	Small group	Observation
		Report on similarities, differences, and best		

		ideas to make one class evaluation sheet	Whole group discussion	Observation and field notes on final activity.
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This activity led to the development of the criteria that students thought should be included in the 15- minute lesson presentation and the criteria that could be explained in the full lesson plan but would not be evident in the presentation time frame. The presentation criteria allowed students to participate in creating the rubric and evaluating their own work. The collaborative effort of the class and teacher resulted in the following criteria for the presentation: clear and correct explanations, clear and correct examples, an engaging activity to include student interaction, a statement of the objective and the goal of the lesson, which was to include relevance and a connection to prior student learning and/or experiences, an engaging hook, and a closure activity. Students planned lessons so that all components matched the objective.

In addition to the presentation, the students would address other aspects of the lesson that were not evident in the presentation due to the limitations of the 15-minute time limit. Plans for addressing cultural responsiveness and differentiation for different ability levels were in the written plans. Students considered the various ability levels and diverse cultural backgrounds and experiences of students and included modifications in their written lesson plans. Students used the evaluation sheet that they created

The daily lesson plans for Day 2 of Phase 2 to are detailed in Table 3.3.

Table 3.3

Daily Lesson Plan for Day Two of Phase 2

Lesson number	Objectives TLWBAT	Instructional strategies	Structure	Assessment
Lesson 5	Evaluate quality criteria for effective teaching in math lessons	<p>Watch three video presentations of math lessons and use the evaluation sheets that were created by the class to evaluate the lessons.</p> <p>Compare thoughts and evaluations of each lesson.</p> <p>Class opinions shared</p>	<p>Individual</p> <p>Small group</p> <p>Whole group discussion</p>	<p>Observation</p> <p>Observations and field notes</p>

The goal of the third phase of the intervention was for students to transition to the work of the teacher who was *practicing pedagogy*. I randomly selected students in groups of three students who would coteach a lesson on either multiplication or division. The purpose of the random grouping was to give students a chance to experience a different role with a group with whom they did not usually work. I hoped to avoid having one person dominating a group. I originally scheduled the videotaping of each presentation for me to review, score, and analyze, but students were adamant that they would be too uncomfortable being videotaped. Because of the time limit that each student had to demonstrate their knowledge of effective teaching practices, PSTs could not demonstrate some of the criteria. For the purpose of determining their abilities of planning for a full

class lesson with support for diverse learners and modifications for different ability-level learners, PSTs completed a written lesson plan that included this criterion.

Each day of the presentations, the students participated as teachers, learners, or evaluators. The evaluators used the evaluation sheets that they had created and with which they had become familiar. At the end of each teaching round, I allowed time for questions and constructive criticism of each lesson. I did not share comments on the evaluation sheet with the presenting groups. This phase of the intervention lasted five days and included two-and-a-half hours of in-class group planning and three days of presentations.

At the end of the intervention, I gave a posttest to assess the growth of the PSTs' CCK. This intervention plan was cyclical, with continual planning-acting-reflecting, then re-planning, and so on, as the plan of action changed.

Data Collection Methods

To collect data that were consistent with qualitative research and appropriate for a case study design, I used multiple methods of data collection. I collected data from observations, pre- and posttests, daily exit slips, a teacher reflective journal, evaluation check sheets, lesson plans, and a lesson plan reflection. I used the initial pretest to ascertain benchmark data for the CCK of PSTs prior to the intervention. After the completion of the intervention, I used a posttest to compare the CCK benchmark data to the post intervention data results to determine if growth in the participants' CCK had occurred

Although I used qualitative data to measure CCK and interpreted it in a qualitative description, I based the evidence of SCK on reflections made during intervals of the intervention. Reflections from the learner at the completion of a lesson, during a lesson as a learner, after developing a lesson as a teacher, and after implementing a lesson as a teacher provided information as to the beliefs the PSTs had of the efficacy of the reformed pedagogy in conceptual teaching as opposed to traditional teaching methods of rote memorization and algorithms. The researcher's daily reflection journal gave insight into the progress of the PST in SCK development and helped me to modify and adjust lessons. Through collaboration with participants in the study, we worked together to create rubric criteria that could be used by the PSTs to evaluate their own presentations and lesson plans, as well as those of their classmates. The rubric criteria indicators were not developed until post presentation when I analyzed and ranked the group presentations as to their quality. Based on the best, middle, and low groups and the levels of SCK that they demonstrated, I inductively developed the final rubric product.

The intervention process from pre-intervention data to rubric development combined planning, action, observation, and reflection. Quality action research connects theory and praxis by balancing action and reflection (Stern, 2014). Through the use of the student and the teacher reflections, combined with active participation in the intervention lesson, these criteria were met.

Research Procedures

The research procedures for this study were completed with ethical decisions infused throughout the research study process and with a systematic, collaborative,

cyclical approach, all of which are characteristic of action research (Herr & Anderson, 2015). To begin the process of study, I applied to the institutional review board (IRB) with details about my intended research procedures and participants. I was granted approval from both the University of South Carolina and at the university where the study took place.

After gaining approval from the institutions to begin the study, I then began to identify volunteers from the group of PSTs who could help me focus on learning the meaning that they held for the problem of practice. I purposefully chose the pool of potential participants from my class of PSTs for elementary mathematics teachers that included a study of rational numbers, which was the focus of the research. In keeping with the ethical considerations of action research, participants had to sign a form consenting to be a participant in the study.

Before the students signed the informed consent form (Appendix A), I explained to them the purpose of my study and the role of the volunteers who chose to participate. I also made clear to each student that there was no pressure to participate and that participation would not affect their grade in any manner. In addition, I advised each student in the class that nonparticipants would share in the same classroom assignments and have the same assessments as the participant volunteers, but the difference would be that their results would not be analyzed or used in the data for the research study. After identifying my participant group of all my 27 students, I later decided to use the data to narrow my participant focus. I chose the nine participants for the more intense analysis based on their performances at the end of an intervention. An intervention is a hallmark

characteristic demanded by action research and one which one distinguishes it from traditional research (Herr & Anderson, 2015).

The collection of data from a teacher-made, four-question assessment on fraction operations (Appendix B) preceded the intervention. I intended to gather benchmark data as to the CCK of the PSTs from these four questions, which I made based on similar released items from a test of mathematical knowledge of teaching designed by Ball, Thames, and Phelps (1988). Following the four-question pretest, PSTs were given a six-question attitudes and beliefs survey to determine the students' levels of confidence in their math abilities and their levels of anxiety in math classes. An open-ended question was also asked about a watermark experience in the students' prior experiences.

The intervention began after I collected the data from the participants. The first phase lasted for three days and I designed it to improve the CCK of the PSTs and to give them opportunities to experience model lessons of learning and teaching math through conceptual methods, instead of procedural methods. Day 1 began with a discussion of the meaning of fractions and their multiple representation through using various manipulatives. A key idea was to show that the size of a fraction depends on the size of the whole, and that two fractions from different wholes cannot be compared.

Using fraction circles, we showed that one whole could be represented by a circle, and $\frac{2}{3}$ of the circle was represented by two parts of the circle that were represented by three equal pieces equivalent to the whole. We then used the pattern blocks and defined a whole as one hexagon. This gave students the opportunity to see that the whole was represented by a different shape than what they had been accustomed to using. To

find $\frac{2}{3}$ of the whole hexagon, the students found that two rhombi from their pattern blocks would represent the fraction. The last representation was with cuisenaire rods, in which we defined a whole with one rectangular blue rod. The fraction $\frac{2}{3}$ was then represented by two green rods. PSTs had the opportunity to see visual representations of one whole and of the fraction $\frac{2}{3}$, leading them to understand the key idea.

Students worked together to find other ways to represent the same fraction using two color chips and number lines. The class then viewed a video of young children representing fractions with manipulatives. The video came from the e-text that is required for the course. Students wrote reflections on what they had learned from the manipulative demonstrations and evaluated the use of each as related to their own learning process as a closure activity for the lesson.

On Day 2, we continued to use the manipulatives to represent equivalent fractions by using the van de Walle (2007) fair sharing activity and a Billstein et al. (2016) activity. PSTs were given options to use manipulatives of their choice to find equivalent fractions, but different groups demonstrated the strategies, which included all types of the manipulatives. On Day 3, the PSTs brought their laptops to class to complete a Desmos online activity to find equivalent fractions. The activity was called Polygraph: Rational Numbers. The second activity was to use fraction strips to find equivalent fractions. A discussion ensued about the need for finding equivalent fractions with a common denominator when fraction addition and subtraction are used. Closure included beliefs about the efficacy of the strategies that had been used for the past three days to increase conceptual understanding. PSTs wrote a reflection based on their own conceptual understanding and the strategies that best served to affect their own CCK.

I designed Days 4 and 5 to give PSTs an opportunity to reflect on pedagogy by defining quality criteria for teaching and evaluating the criteria that they observed in the lesson of others. This was the beginning of the development of their SCK needed for teaching mathematics to their future students. On Day 4, PSTs were assigned to small groups to brainstorm criteria for quality teaching that they had observed from prior experiences. Through group consensus, they were to compile a list and post it on chart paper for a class gallery walk. After all lists were posted, groups traveled together around the room to discuss the thoughts posted by each group. PSTs each had sticky notes that they could use to comment on the work of others and on which they could make notes for their own group. At the completion of the exercise, the whole class shared what they saw and came to consensus about what criteria should be included on an observation checklist that they could use as a tool for evaluating the lessons of others. In addition, the group decided on important characteristics that I should observe in the 15-minute presentation that they were to do in Phase 3. By sharing these ideas, the group collaborated with me to develop an evaluation tool (Appendix D) that they could use an artifact that was useful for beginning development for a rubric in the last phase of the intervention.

On Day 5, the class watched several videos that I had chosen for them to evaluate for quality teaching criteria. I chose videos of lessons that represented a wide range of effective demonstrations of teaching. Students used the observation checklists they developed to make evaluations. In small groups they shared their thoughts and then communicated them in a whole group discussion. At the end of the class, students counted off by nine to form nine groups of three people. I chose this random method for

forming groups so that participants could experience working with different people and would share the workload evenly with their newly formed partners.

On Days 6 and 7, the groups met to plan a lesson on either multiplication or division of fractions using methods of conceptual understanding. I facilitated the group discussions and answered questions when I was needed. At the end of the class, the groups had to turn in a progress report to update me on what they had chosen to teach and what ideas they had decided on for their presentations and in their written lesson plans. Students met outside the class time to complete tasks not finished in the time allotted for class. This work continued on Day 7 with students completing their written lesson plans and dividing up responsibilities for each person to have equal teaching time during the presentation.

Days 8, 9, and 10 were divided so that three groups per day could present their lessons, with time between lessons for constructive feedback to each presenting group. Each day, every PST had an assignment as a teacher, a learner, or an evaluator. The groups had previously drawn numbers and had received an assignment chart to tell them their role for each day (Appendix F). Teachers were the three-member groups, while nine learners sat in a semicircular arrangement to participate in the lessons that were presented by the three different groups. The evaluators used the observation checklists with which they were familiar to evaluate each group's teaching presentation. Two days following the intervention period, students submitted their written lesson plans and reflections on their presentations and took a posttest of the four questions to assess changes in their CCK.

Students worked in small groups throughout the intervention phases to give PSTs opportunities to communicate mathematical ideas and reflect with small groups on the meaning of the learning each day. Working in cooperative groups to collaborate as a team is a part of the reformed pedagogy strategies used to develop confidence in mathematical skills and to build intrinsic motivation to value these strategies. In all phases of the intervention, the emphasis of teaching mathematics was on teaching for conceptual understanding, rather than for procedural knowledge.

Treatment, Processing, and Analysis of Data

After the intervention, I changed roles from teacher to researcher. My first task was to collect and analyze data from the pretest and posttests for CCK to ascertain the effectiveness of the intervention for CCK development. To assess the effectiveness of the intervention for SCK development and teacher beliefs about math instruction, I worked with a priori coding using a researcher/practitioner designed list of criteria for SCK and teacher beliefs about mathematics instruction to analyze all students' progress and to compare individuals to the whole class. Using Likert scale responses and open-ended responses, I analyzed the pre- and posttest intervention survey questions.

I reported all of the data collected from pre/posttests and surveys quantitatively in tables, as well as the rubric scores from the lesson plans, but I translated the results from all of the tables to support a qualitative analysis of the results. Since students were only teaching other preservice students, some rubric criteria could not be met with the presentations but could be met in lesson plans.

The reflection journal and the exit slips provided qualitative data that supported the summary and conclusions of the study, which I reported in a rich description of the study conclusions. I addressed each component of the criteria for SCK within the class, and I evaluated each criterion during different parts of the study.

The data collected was connected to quality criteria through member checking, collaborative engagement of various stakeholders, triangulation of data resources, and a rich description of CCK, SCK, and conceptual understanding for study participants. Quality action research includes collaborative, participatory involvement of people concerned in the research process, and also agreement upon ethical rules for the collaboration (Stern, 2014). The research study involved member checking to ensure the accuracy of the findings, and collaborative engagement with other instructors of the course at the university to help with reflection of the findings and to help identify any biases. I considered the perspectives of various stakeholders and used triangulation as a major component of this design. This study used several methods to collect data, including surveys, reflections, artifacts, and observations. The triangulation of the data helped me to more fully understand the constructs being addressed in the study. A rich description of each construct of the study was in place to ensure inter-rater reliability. The ethical standards were achieved through the anonymity of participants and a full disclosure of study expectations from the onset of the research process.

Summary

The purpose of this research study was to examine ways to support my student PSTs in improving procedural and conceptual math content mastery, as well as the

specialized content knowledge that they will need in order to feel empowered to teach their future students. The evidence collected through results of skills assessments in my classes led me to choose multiplication and division of fractions as a focus for this study. Teacher beliefs about the efficacy of conceptual methods of teaching mathematics versus procedural teaching methods needed to be developed.

The research design was a qualitative case study chosen for the purpose of exploring one particular class of PST at a university. I addressed quality criteria for the design in this study which supported the theoretical framework on which the study was based. Mathematical knowledge of CCK and SCK, reformed pedagogy, and culturally responsive pedagogy were the theoretical foundations on which the study was built, and which promoted understanding of the authentic and worthwhile problem of the research.

The participants in the research were a purposeful sample of university students who volunteered to become involved in the study. The data collection methods were consistent with a qualitative research study and involved multiple sources of data measures. I provided details of the intervention, data collection, and data analysis in this chapter, all of which supported the theoretical framework, the problem of practice, and the research questions. I made efforts throughout the study to ensure that ethical considerations were in compliance to protect the well-being and interest of my study participants. Quality criteria for qualitative studies had confirmability that the study's findings were the results of the experiences of the informants rather than the preferences of the researcher and can be achieved through an audit trail of the raw data, memos, notes, data reduction, and analysis (Guba & Lincoln, 2005). The concrete evidence listed in the criteria is available to confirm the study findings.

Chapter 4

Presentation and Analysis of Data

Research in mathematics teacher education and my own experiences as a math teacher educator have indicated to me that undergraduate preservice math teachers have not been provided with a solid background in conceptual mathematics. This common occurrence leads to significant challenges for these students as they both learn (Shulman, 1986) and learn to teach (Ball, 1993) mathematics. Based on these experiences, the purpose of this study is to better understand how instructional strategies that promote the development of conceptual understanding of mathematics among preservice elementary mathematics teachers leads to deeper conceptual understanding of mathematics and how the views of these PSTs on teaching for conceptual understanding change as a result of these efforts.

In this study, I enacted several instructional strategies in an attempt to increase the common content knowledge (CKT) and the specialized content knowledge (SCK) of preservice teachers (PST). I measured the effectiveness of the enactment of these strategies, the impact of these strategies on the intrinsic motivation of PSTs to value conceptual mathematical understanding over a procedural understanding of mathematics, and the PSTs' motivation to teach for conceptual mathematics understanding after the intervention. I then designed the intervention from the perspective that quality math teacher education should provide opportunities for PSTs to develop both conceptual and

specialized understanding of the content they will teach and become familiar with instructional strategies that will foster equitable learning for all students (Ball, 2003; Smith, 2013; Ladson-Billings, 2009). The primary research question that guided the data collection was:

1. What are the important factors to consider when developing instructional strategies that promote specialized content knowledge and the intrinsic motivation to teach for conceptual understanding among preservice elementary mathematics teachers?

This question was addressed through an action research qualitative case study design that included a three-phase intervention plan. Phase 1 was the experiencing pedagogy phase, where the PSTs were learners and the primary function of the phase was to increase the CCK of the PSTs. Phase 2 was the reflecting on pedagogy phase, in which students were beginning to develop SCK and beginning to make meaning of their own learning. Phase 3 of the intervention was the practicing pedagogy phase, where PSTs participated as practitioners planning and implementing a lesson. In this phase, PSTs gave evidence of their SCK gained during the intervention, and an analysis of their work led to the development of an inductive rubric to assess the classwork and to form instruction for my future work as a practitioner. This chapter provides a detailed description of the various data collected during the study, the results of my analysis, and my interpretations of these results. To find meaning from the intervention methods, I used multiple data collection methods to gather information. Classroom observation of activities and student discourse, exit slips, student journal entries, and teacher field notes gave me data to help provide a rich description of student progress. I used results from pretest and posttest data to analyze the development of CCK and used an attitudes and

beliefs survey to measure the confidence and anxiety levels of students about learning and teaching mathematics.

Preintervention Data

In order to measure the impact of the intervention on PSTs' CCK and SCK, I needed to identify the prior knowledge and dispositions towards mathematics education held by the PSTs who participated in this study. I collected this data from PSTs through the use of a conceptual mathematics pretest and a digital questionnaire that focused on their attitudes and beliefs about mathematics education. In the following section, I provide a description of the data, my analysis of this data, and discuss the impact my analysis on the subsequent selection of instructional strategies that would foster the development of CCK and SCK among the PSTs.

Common Content Knowledge Prior to the Intervention

The pretest was primarily used for benchmark scores of CCK of fractions held by the participants at the onset of the research study. The four-question assessment consisted of word problems that would require CCK of the four operations of arithmetic on fractions and the comparison of the values of fractions. The results on the pretest for 27 students are included in Table 4.1 as a data table and in Table 4.2 as a Pie Chart.

Table 4.1

Number of Correct Responses to Pretest Questions

Question	Concept tested	# correct answers
1	Fraction multiplication	10
2	Fraction addition and subtraction	7
3	Fraction value comparison	20
4	Fraction division	12

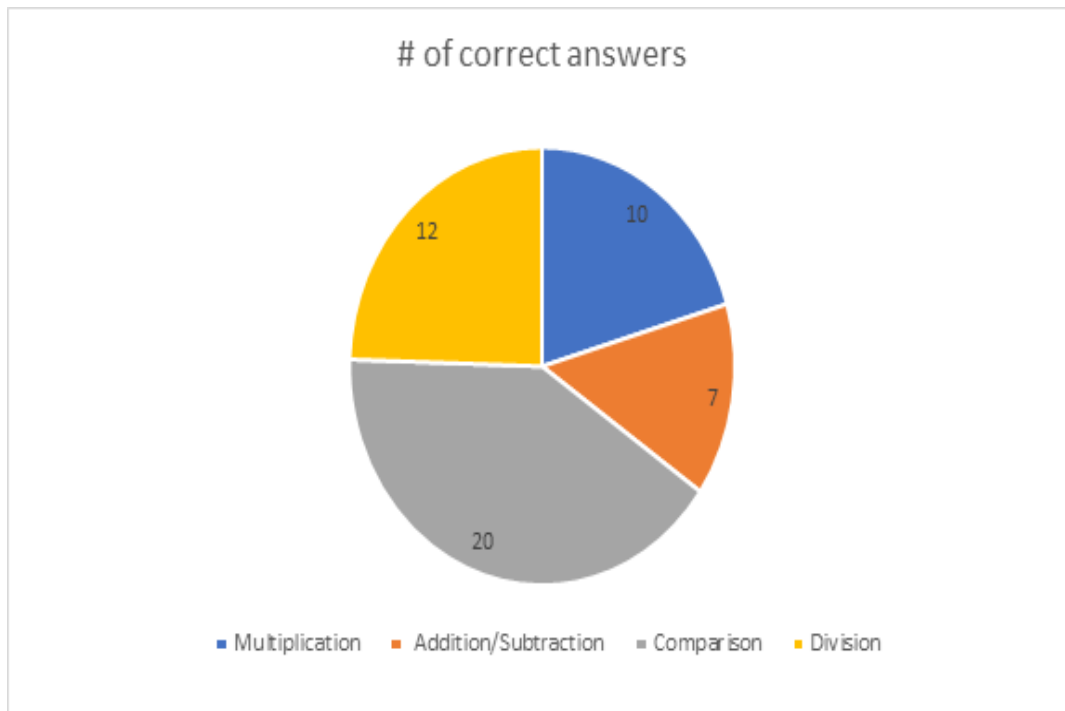


Figure 4.1. Visual representation of correct pretest responses.

The whole group class scores were assigned on a scale of 25 points for each correct answer. Figure 4.2 shows that the majority of students scored a 25 or a 50 on the test.

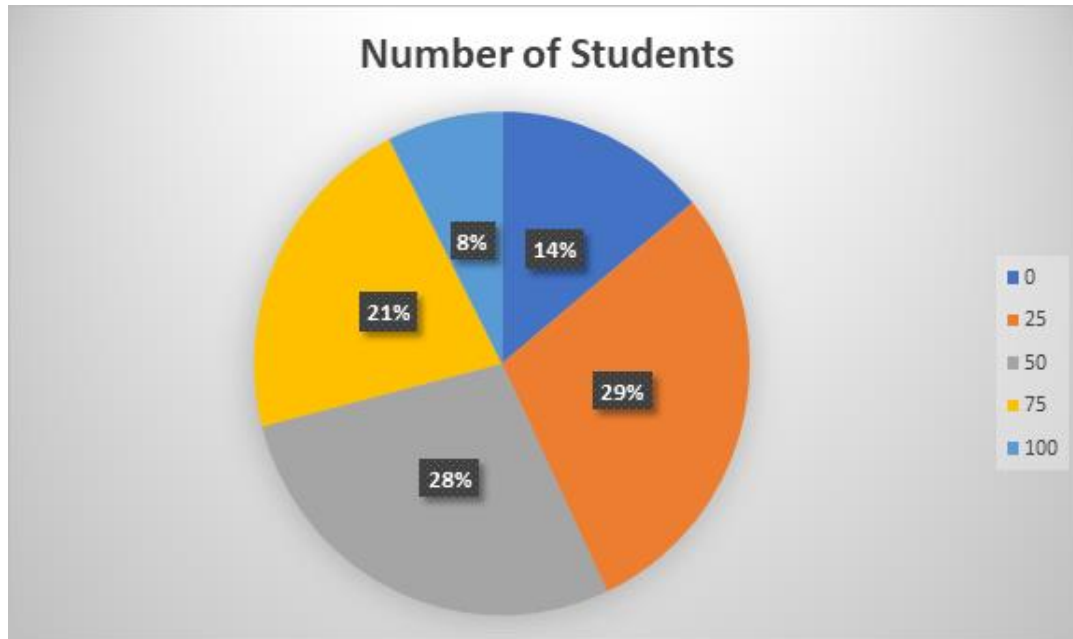


Figure 4.2. Percentage of students and their pretest scores.

Question 3 had the most correct responses and involved finding the larger of two fractions with a common numerator, $995/8432$ and $995/8429$. I intentionally constructed the question with large numbers for the numerators and denominators to discourage students from converting the fractions to decimals or using the method of cross multiplication to determine the answer. Students could use reasoning to find the answer since the numerators were the same value. I considered students who responded correctly and gave a valid explanation of their reasoning to be competent in this area of CCK.

The question that resulted in the most incorrect responses was Question 2:

Mr. Smith's will provides that his five children will share his estate according to the following provisions. Al receives $\frac{1}{3}$ of the inheritance, Bob receives $\frac{1}{4}$, Cal receives $\frac{1}{5}$, and Don receives $\frac{1}{6}$. What fractional part of Mr. Smith's estate does Ed receive?

To be considered competent in addition and subtraction of fractions, the student had to first understand that the problem called for adding the fractional parts already assigned, then find equivalent values of the fractions using a common denominator. Next the student would need to find an equivalent fraction name for one and subtract the sum already assigned to the other four children. Most students missed this problem because of their inability to correctly identify the correct arithmetic operations necessary to solve the problem. Only five students did not use a common denominator, and two students missed the question through an error that would be considered as careless arithmetic.

More than half of the students in the class also missed Questions 1 and 4.

Question 1 was:

Plans for a new park show that $\frac{3}{5}$ of the park will be for a playground. Of the designated playground area, $\frac{1}{4}$ will be reserved for special needs children and families. What fraction of the total new park will be a playground for special needs children and families?

PSTs scored a competent mark for CCK on this problem if they could identify that the problem required multiplication of fractions and correctly find the solution. A possible explanation for the reasoning involved in the problem would be a pictorial representation

of the park with the areas appropriately partitioned by the correct fractional pieces. Errors on this question included: omission of the problem entirely, use of the incorrect arithmetic operation, and incorrectly multiplying the fractions. Mistakes involving multiplication facts occurred with three students.

Question 4 was:

Jane is making apple turnovers. If she uses $\frac{3}{4}$ of an apple for each turnover, how many turnovers can she make with 18 apples?

Some students immediately recognized that the problem could be solved by taking the 18 apples and dividing them into $\frac{3}{4}$ apples per turnover. Others correctly solved the problem conceptually by drawing a representation of the 18 pies and counting the number of $\frac{3}{4}$ that could be obtained. Incorrect responses were the result of incorrect interpretation of the problem leading to use of the wrong arithmetic operation, incorrect procedures for division of fractions, or errors in division facts.

Attitudes and Beliefs about Mathematics Education

Two weeks prior to the intervention, I collected other pre-intervention data information from an attitudes and beliefs survey. The students rated six statements to assess the confidence levels and anxiety issues of the PSTs in math class. The answer choices were *strongly agree*, *agree*, *neutral*, *disagree*, and *strongly disagree*. These responses were converted to a 5-point Likert scale to give the researcher a way of making meaning of the data.

Statement 1 corresponded to confidence levels. “I feel confident in my math abilities” was rated with twice as many negative responses as positive responses. There were 67% responding with some degree of disagreement and 33% with some degree of agreement. Using the Likert scale with 5 (*strongly agree*), 4 (*agree*), 3 (*neutral*), 2 (*disagree*), and 1 (*strongly disagree*), the mean score was 3.2 and the mode was 2.

Statements 2, 3, 4, and 6 were all related to issues of anxiety in math classes. For Statement 2, “I get nervous before math tests,” 87% reported agreeing and 13% disagreeing, with a mean of 4.2, and a mode of 4. Statement 3—“I draw a blank during math tests, even though I am well prepared beforehand”—had 74% agree, 26% disagree, a mean of 3.8, and a mode of 4. Statement 4—“I am anxious when I feel like the teacher is going to call on me”—had 67% agree, 33% disagree, a mean score of 3.7, and a mode of 5. Statement 6—“I am comfortable answering questions in math class”—required oral clarification to differentiate it from Statement 4. Statement 6 refers to volunteering a response, rather than being asked for a response. The results were 33% agree, 67% disagree, mean score of 3.1, and a mode of 4. Statement 5 was about math abilities of different gender groups. The statement was “Boys are better at math than girls.” All students disagreed with the statement, with the exception of one female student. This resulted in the following statistics for the statement: 3% agreed, 97% disagreed, with a mean of 2, and a mode of 2.

Interpretations of Both Data Sets

The results of the pretest for CCK showed that PSTs needed additional support in developing CCK, particularly in conceptual understanding of the meanings of the

operations. Indications from the analysis of the pretest pointed to a lack of understanding the *when* and *why* of using the operations on fractions rather than the *how* to use them.

Low confidence levels in and math anxiety issues were prevalent among the PSTs who participated in the attitudes and beliefs survey. Developing a more positive attitude about personal abilities in math was paramount in decreasing anxiety issues and increasing intrinsic motivation to study math.

Impact on the Design of the Intervention

The results of the combined pre-intervention data led me to include more cooperative learning as an intervention strategy so that students would not only learn from each other but also gain confidence from the small group discourse and support from classmates. I also decided to use meaningful problem solving at this point so that PSTs could see the relevancy in the mathematics being presented and the value in the methods in which it was taught.

The pretest for CCK convinced me of the need to teach using strategies that would increase the conceptual understanding of the PSTs in my class. After analyzing the student responses on the pretest, I concluded that the concept of how to perform operations on fractions was not an issue in most cases. To develop the PSTs' understanding of the *when* and *why* of fraction operations, it was clear that the intervention activities must focus on the concrete stage of learning math concepts and that using multiple representations on these concepts was necessary to increase their CCK.

The literature on the efficacy of using concrete strategies to increase conceptual understanding guided my decision to begin the intervention, and this reality confirmed a clear next step in the iterative cycle of action research, plan–act–observe–reflect (Herr & Anderson, 2015). A plan for a three-phase intervention had been established and was implemented after the reflection of pre-intervention data confirmed that PSTs in my class needed the intervention support to increase their CCK and SCK.

Intervention Phase 1

As students worked in collaboration with others, they had the opportunity to share their ideas or to learn from the ideas of others. I planned this strategy to give every student a chance to think about answers before they were given and to give students confidence in communicating their mathematical ideas. Collaboration is a key component of reformed pedagogy (Smith, 2013) and culturally responsive teaching (Ladson-Billings, 2009), both of which are part of the theoretical framework of the study. Discourse among students became a vital part of helping students to make convincing arguments, to analyze other strategies, and to evaluate the different approaches of others in the class. I chose the strategy of student collaboration to help PSTs to become more confident in their math abilities and to relieve their anxiety about answering aloud in math class. In order to give students an opportunity to choose an appropriate strategy and to hold them responsible for checking the reasonableness of their answer in the context of the problem, I purposefully selected problem- solving tasks. Communication of individual ideas to classmates helped the PSTs understand that multiple strategies are acceptable in problem solving and gave them motivation to apply their CCK to new situations. Table 4.2 shows the daily lesson plan outline for this phase of the intervention.

I recorded observations daily in my reflection log during this phase of the intervention. Exit slips and student reflections about the daily lessons gave me insight into students' attitudes, beliefs, and perceptions about the strategies used during the learning phase. I coded the comments on concrete methods using different manipulatives using the Saldana (2013) coding process for qualitative research. For the initial coding I looked for a variety of commonly used phrases and words that gave insight into the research questions. I then recorded the first impressions with a priori goals in mind to enable me to answer the research questions. As my first goal of the initial coding, I decided to break the data into parts and then to look for similarities and differences in the data. During this coding cycle, the impressions came from journal reflections, exit slips, and comments that students made during observations. At this point, there was no sequence or logical grouping of the data. Table 4.2 shows the comments lifted directly from student reflections and conversations on the left side with the initial coding on the right side.

Table 4.2

Initial Coding Impressions

Verbatim Student Comment	Initial Coding
I never realized how many options there are for representing an equal value	Representations of equal values of wholes
I never thought of trapezoids, hexagons, and triangles as being parts of a whole.	Different shapes represent fractions.
I have learned to make fractions out of different shapes.	Different shapes represent fractions.
There are multiple ways to represent wholes.	Multiple representations are possible.
I had never thought of fractions as pieces.	Fractions are pieces.

The size of the fraction depends on the size of the whole.	Importance of the size of the whole
Fractions are the shares that the whole is split into	Fractions as shares
I had always thought of fractions as parts of circles, but fractions can come from all shapes.	Fractions represented by many shapes.
I have been more active in learning with these lessons.	Active engagement
I am capable of understanding and teaching fractions this way.	Growth in confidence
Cuisenaire rods help me to see the parts easier	Seeing the parts
Small white Cuisenaire rods helped me to compare parts of the whole and to think of parts in common terms	Comparing parts and finding common terms to explain different fractional parts
My internship students are using the Cuisenaire rods and like them	Young students enjoy using Cuisenaire rods
Cuisenaire rods are versatile.	Versatility of rods
I think Cuisenaire rods are too abstract.	Cuisenaire rods are abstract.
I can visualize the whole with pattern blocks.	Visualize the whole
Pattern blocks are easy for me to use.	Ease of use
I like to overlay the fraction circle to compare them.	Overlay fraction circles to compare
Fraction circles help me see how fractions work together.	See how fractions work together
I like seeing the reasoning behind something.	Conceptual
Manipulatives leave less room for error than procedures.	Conceptual over procedural
Better than using paper	Conceptual over procedural

I rearranged and regrouped these comments and codes multiple times in hopes of finding similarities, differences, or connections. Each cycle of coding led to a streamlining of the categories before I made a decision to use three main coding themes:

representations of fractions, means of engagement, and conceptual understanding and learning. Each of these codes are more thoroughly explored in the interpretation of the data findings section.

Table 4.3

Comparison of Correct Questions for Pretest and Posttest

Fraction Concept	Pretest number correct	Posttest number correct
Multiplying	10	22
Add/ subtract	7	11
Comparing	20	20
Dividing	12	19

Interpretations of Phase 1

When I began looking at the data gathered from the individual students, I started the investigation with the CCK knowledge from the pretest and the posttest. I analyzed this data to help provide answers to the Research Question : “What are the important factors to consider when developing instructional strategies that promote specialized content knowledge and the intrinsic motivation to teach for conceptual understanding among preservice elementary mathematics teachers?” The lack of CCK among my PSTs was an important factor in planning the next steps for supporting them in developing the SCK necessary for quality teaching practices. From my observations, I concluded that the PSTs did not have the knowledge of the subject matter that they needed to teach, a requirement for knowing the best ways to teach it. The data included in this section gives a clear indication for the necessity of the intervention and the effectiveness of the intervention

stage for improving the CCK of the research study participants. The whole class data is presented first in Table 4.4.

The gains for each concept questioned were most substantial for the multiplication and division concepts, which were the focus of the third intervention stage, *practicing pedagogy*. The addition and subtraction concepts had been studied prior to the intervention, but only the step for finding equivalent fractions with common denominators was a part of the *experiencing pedagogy* included in Phase 1. The gain made by the class on this concept was 15%. The comparison of fractions concept remained high but showed no change. Individuals also made substantial gains in scores as evidence in Table 4.4 shows.

Table 4.4

Comparison of PST Scores on the Pretest and Posttest

Score	# of students from pretest results	# of students from posttest results
100	2	7
75	6	9
50	7	7
25	8	2
0	4	2
Mean	44.4	65.7
Mode	25	75

In an item analysis of the posttest questions, I saw that of the seven students who missed the first question, three had incorrectly drawn a model for the picture to represent the multiplication problem, three students had chosen the operation of subtraction as their

solution method instead of multiplication, and one student offered no work or answer. On question two, there were eight students who could not correctly identify that first addition, then subtraction was necessary to solve the problem, three students had forgotten to do the last step, but had complete 95% of the problem correctly, three had made a careless error with addition of whole numbers, and one student had simply incorrectly guessed an answer with no reasoning shown. On question three, comparing fractions, four students incorrectly guessed an answer, while the other three incorrectly explained their reasoning. Question four pertained to division of fractions. Three students only used $\frac{3}{4}$ of each apple and ignored the $\frac{1}{4}$ apple left over from each whole and did not combine them to make another whole. Two students incorrectly tried to use a procedure only and switched the dividend and divisor. The other three students used multiplication instead of division to find their incorrect solution.

At the conclusion of the three intervention stages, I noticed a significant difference in the CCK and SCK growth among the different groups. I then ranked the groups from highest to lowest performance. The difference in the quality of the presentations led me to look more closely at the full intervention picture of the three students in the best group, the three students in the middle group, and the three students in the weakest group. From the three groups I hoped to gain insight into the wide range of abilities and understandings from students who had just experienced the same intervention. By interpreting the data results of the nine members, I attempted to answer the question, “Why did this disparity in student knowledge happen?” A discussion of the individual performances of the selected PSTs based on data reflection from Phase 1 and its effectiveness to address the CCK deficiencies in the PST participants is necessary to

find these answers. The whole group data is displayed in table 4.5, with the high, middle, and low groups listed in bold print.

Table 4.5 *Comparison Scores from All Groups*

Pseudonym	Pretest Score	Posttest Score	Percent of growth	Pass (Y/N)
Ann	50	100	100%	Y
Maggie	25	100	300%	Y
Kate	25	75	200%	Y
Rina	75	75	0%	Y
Jay	50	50	0%	N
Charlie	0	50	50%	N
Vanna	50	50	0%	N
Mary	25	25	0%	N
Cheryl	100	100	0%	Y
Carrie	100	75	-25%	Y
Ashton	25	100	300%	Y
Kenny	0	50	50%	N
Reagan	0	0	0%	N
Cat	25	100	300%	Y
Alison	50	50	0%	N

Niland	25	25	0%	N
Mandy	75	100	33%	Y
Melly	75	75	0%	Y
Blake	75	100	33%	Y
Candy	25	50	100%	N
Bobby Ann	25	50	100%	N
Jana	75	75	0%	Y
Alex	0	100	400%	Y
Kia	50	75	50%	Y
Molly	50	50	0%	N
Amy	50	50	0%	N
Joy	50	75	50%	Y

The highest performing group included Ann, Maggie, and Kate. Their pretest and posttest data are shown in Table 4.7 and gives an indication of their development of CCK.

Table 4.6

Comparison Scores from Top Group

Student	Pretest score	Posttest score	% of increase or decrease
Ann	50	100	100%
Maggie	25	100	300%
Kate	25	75	200%

From data gathered on these three students, it is evident that the CCK significantly improved since the students were involved in the intervention process and that all three students could be classified as proficient in CCK.

For the pre-intervention survey about beliefs and attitudes toward teaching and learning math, I saw that Ann was confident about her math abilities at the beginning of the research study. She had no anxiety issue, other than she strongly agreed that she was nervous when the teacher called on her. Ann said that her confidence came from encouragement from her mother.

Maggie was not confident in her math abilities and agreed that testing made her nervous, even though she prepared for them beforehand. She commented that she prefers working alone and enjoyed worksheets more than manipulatives. Maggie's survey revealed surprising information since she became the most confident member of the best-performing group. Maggie had commented to me that she once had a teacher tell her that she was bad at math, and she had believed it since that day.

Kate did not perform as well as the other two group members on CCK knowledge. She did well on the multiplication problem and the fraction comparison problem. She omitted the addition and subtraction problem, and she missed the division problem because she thought it was representative of multiplication. Her survey responses were that she had neutral feelings about her math abilities and her feelings about the teacher calling on her. She did respond that she was anxious before and during testing. She claims to have conquered her fears of answering aloud in class when she was asked to show the class how she had successfully completed a certain problem on a test.

From the journal responses, all three students responded that their favorite manipulative for learning about equivalent fractions was the pattern blocks. Kate and Maggie each commented that they liked the hands-on approach of using them and felt that young children would respond well to the pattern blocks as a learning tool. Ann liked the pattern blocks to complete the activity because of the visual aspects. She wrote that she could visualize the whole and the pieces that make up each fractional part of it with this manipulative.

Ann did not care to use the fraction circles for learning fractions because there were so many pieces. Both Kate and Maggie saw benefits of using the fraction circles, and Maggie commented that she thought a benefit of this manipulative was to see how fractions work together. All three women agreed that the Cuisenaire rod was their least favorite tool and felt that this manipulative had been the least effective in developing their conceptual understanding of fractions.

Table 4.8 contains the data from the pretest and posttest to assess CCK development for the group ranked in the middle of the overall class performance.

Table 4.8

Comparison Scores for Middle Group

Student	Pretest score	Posttest score	% of increase or decrease
Reagan	0	0	0%
Cat	25	100	300%
Alison	50	50	0%

Initial impressions were that this group had little CCK at the onset of the research and made no improvements post-intervention. The one exception was Cat, who scored 100% on the posttest and made a 300% gain. She is the only member of this group to gain CCK proficiency from the intervention. The question that follows this information for the researcher is: “What was the reason for the marked difference in the CCK development for these three students?”

Information from the first exit slip showed that Cat felt the activity with the Cuisenaire rods gave her an “Aha” moment. She found that the smallest unit, or the white rods, could be used to find the size of all the other rods. She felt empowered by this discovery and said that she believed that she could teach with the rods. Alison simply wrote that she had learned to explain how she compared her fractions using the manipulatives, but she did not give any specific details about which manipulatives

worked best and how they had helped her to compare. Reagan wrote that she had not learned anything new from the lessons.

Alison stated in her journal writings that she really enjoyed learning with pattern blocks, fraction circles, and Cuisenaire rods. The only one with which she had previously been familiar was the pattern blocks, which were used by students in her field experience classes. She was willing to try the other manipulatives with the children, but the school did not have those materials. Reagan commented that she thought that all the manipulatives were beneficial because they were good for tactile and visual learners. Cat said that she looked forward to the pattern block homework because she enjoyed making sense of the problems. She liked the meanings that the fraction circles brought to the lesson but thought that they were too thin for little children to handle easily. She did not enjoy using the Cuisenaire rods and found them confusing. Cat was the only student to mention practicing the strategies through the independent assignments given for homework, and she was the only one who was able to give a specific example of how the manipulatives were helpful to her.

The evidence from the attitudes and beliefs survey clearly showed that each of the students had anxiety issues about math tests and answering in math class. None of the women were confident in their math abilities. In the reflection of their presentation, Cat wrote that the group worked well together, and Reagan said that she had learned something from the others. Cat also expressed regret that she and Reagan had not switched roles, since Reagan was not as familiar as she on the section she taught. This statement by Cat led me to conclude that Cat was confident, comfortable, and intrinsically motivated to use the strategies for conceptual understanding

Evidence from the pretest and posttest results of the CCK suggests that the three students from the weakest group failed to make much improvement from the beginning of the research study throughout the course of the intervention. Although these three students did not show growth in their CCK knowledge, their scores were equal to or higher than two of the three students in the middle group. The results for the three members of this group can be found in Table 4.9.

Table 4.9

Comparison Scores for Low Group

Student	Pretest score	Posttest score	% of increase or decrease
Molly	50	50	0%
Amy	50	50	0%
Joy	50	75	50%

Each of the students in this group expressed a lack of confidence in their math abilities and their anxieties about math learning in both testing and in class participation. Joy commented in one reflection that she once had a teacher to tell her that math wasn't for everybody. Amy wrote in her reflections that she had been in math classes where the primary instruction was to copy rules and then do book work. Joy and Molly both wrote in their journals that they felt more confident in their abilities to understand and work with fractions after the Phase 1 intervention lessons, but Amy had no comments to make on the lessons.

Amy did have positive thoughts that she shared about using the manipulatives. She shared that all the manipulatives were good for her learning style as a visual learner.

Joy thought that all the manipulatives helped her to learn more about fractions because they were hands-on. Molly had several insightful comments that revealed her deeper understanding of the concepts: “I always thought of fractions as parts of a circle, but pattern blocks showed me that they were parts of all kinds of shapes ... I was able to compare fractions to see how close they are, such as $1/11$ and $1/12$ by stacking the fraction circles.” This helped Molly to realize that if the numerators are the same, the number of pieces is the same, but the more equal pieces into which the denominator is split, the smaller the pieces are. This was the conceptual explanation for Question 3 on the CCK test. Molly also said, “Manipulatives turn abstract problems into concrete problems” and “Manipulatives leave less room for error than procedures do.”

Molly’s failure to improve on the CCK for the posttest is puzzling, since she appears to grasp the conceptual understanding of the lessons. Looking more closely at the pretest and posttest for CCK, I could see her score on each would have been a 75 except for a careless counting error she made on each test. Ironically, her mistakes were on the multiplication and division problems, the two operations that she had just studied, and one of which she helped teach to her classmates.

Intervention Phase 2

This phase in the intervention was useful in building confidence in individual math abilities and in sharing ideas in class. The opportunities were always structured for students to use think–pair–share, or think individually, collaborate with a partner, and then share thoughts and comments in small groups or whole groups. No one had to give a response without first consulting with at least one classmate.

By using the evaluation sheet that the class had previously developed, students had opportunities to evaluate lessons from videos that included teaching for conceptual understanding. Conversations were ongoing as to the methods observed in the videos that portrayed teaching for conceptual understanding rather than procedural strategies. Evaluations of the lessons allowed students opportunities to buy-in to this reformed pedagogy and become intrinsically motivated to teach using these methods. Students asked me why they were never taught that way (reformed pedagogy). Some of the PSTs commented that they would have enjoyed math and understood it better if these methods had been a part of their learning experiences.

This phase was constructivist in nature, as it gave students the opportunity to create meaning from their own learning. Collaborative construction of knowledge through collaboration with other students and reflection of teaching practices allowed students to participate in a metacognitive process to effectively address the purpose of the intervention phase. The constructivist design of the lesson was purposeful, and lessons were designed to have students synthesize the components of SCK.

Intervention Phase 3

It was evident from reading the individual evaluations that PSTs were much more lenient with their peers than they had been when watching the videos in class. On some evaluation papers, PSTs checked all criteria as evident in their classmates' lesson presentations and only positive comments were made. On the teacher reflections, I never observed this degree of perfection in even the best presentations.

I chose the highest ranked group as the best group for the quality presentation and lesson plan that they developed. Their lesson on multiplication of fractions engaged learners in a station activity that gave opportunities to use three different types of manipulatives in solving problems. Each group member posed a problem to a group of three learners using one of the types of previously evaluated manipulatives. The PSTs, who were learners for the presentation, solved the problem with a demonstration of the conceptual meaning of the task and without pencil and paper procedures. Each station involved multiple parts so that learners who needed assistance from the student presenter on the first part would have several opportunities to complete a part independently. The group used a unique idea of bringing their station activity to the next group of learners, rather than having the learners move to a new station. Each group member did well in explaining their problem, demonstrating the use of the manipulative to show conceptual meaning of the problem, and assisting learners to successfully complete the activity.

Other strengths of the group were their opening and closure components. The opening statement included PowerPoint slides that clearly stated the objective of the lesson and a hook that related the objective to the number of souvenir bricks taken without permission from the university's Scholar's Walk. This group was one of the few groups that did a warm up that made real connections between the lesson concepts and to prior experiences of the students. The group members verbalized the connections and wrote them on a slide. The closure activity was called 3-2-1. Students were instructed to write an exit slip telling three things they had learned, two things they wanted to know more about, and one thing that they still had a question about. All parts of their lesson matched the objective and I noted no mathematical errors in their work. The lesson was

well received by classmates, who commented on the good feedback, verbal praise, and encouragement that the PST presenters gave to their learning groups.

The group ranked as in the middle of the class performance had some positive moments in their presentation and some that needed a great deal of improvement. The strengths of this group's lesson were that the standard and goals were stated, and that Cat made a connection to prior learning. The example that this teaching group used to support the lesson on multiplication of fractions involved baking sugar cookies. This problem was interesting to the group and Reagan explained it with the use of a pictorial representation. Reagan did make an arithmetic error in her solution, but Cat prompted her to change it. The overall feeling of the lesson was that it was a mediocre explanation and example, but students were not ever actively engaged nor was there ever any activity to promote conceptual understanding. The pictorial representation would have promoted learning conceptually if the solution had been correct and if students could have been actively engaged. Students were instructed to make up a problem and draw a matching picture for the summary activity to close the lesson. The closure activity was weak in purpose or clarity of instruction.

The poorest performance was not difficult to rate. Amy began the lesson for her group by stating the objective and standard for division of fractions. After this point, the presentation was not on target for the criteria. It was evident to me that the members of this group knew what was expected of them, but they could not effectively plan and implement appropriate explanations, examples, or an activity to present the lesson. To engage the learner in the fraction division lesson, the hook was, "How many of you like chocolate candy?" This supposedly would lead into an activity in which students divided

candy bars into fractional serving sizes. However, the activity did not match this objective nor was it correct. The problem presented in the first example was: “If I have $\frac{1}{2}$ of a candy bar and give you $\frac{1}{4}$, what part is left?” This problem is ambiguous as to the meaning of $\frac{1}{4}$. Is it $\frac{1}{4}$ of the $\frac{1}{2}$ that I have? Or is it $\frac{1}{4}$ of the whole candy bar? In either case the problem was not representative of the division problem $\frac{1}{2} \div \frac{1}{4}$. Subsequent problems were equally ambiguous and not representative of the problems presented.

The second part of the presentation was the playing of a rap song that constantly repeated the lyrics, “keep, change, flip.” This trick to remember the procedure for inverting and multiplying did nothing to help students gain any conceptual understanding of when and why the procedure is used.

I was surprised that the presentation missed the point by such a wide margin. Some of the students in the group had some CCK and obviously knew what was expected to satisfy the rubric components. The problem was that they had no clue as to how to match the CCK and SCK. Molly made some good reflections in Phase 1 of CCK development, but she was not the dominant personality in the group. With her lack of confidence in her math abilities, she may have expressed her ideas but acquiesced to the decisions of the dominant group member.

Development of the Rubric

Using the best group as the benchmark of proficiency, I then began to develop a more appropriate rubric than the one originally created. The rubric columns were Exemplary, Proficient, Intermediate, and Emerging. The benchmark group was used as a model for the Proficient column. The Exemplary column was based on raising the bar for

the Proficient group and used indicators that would be evident if the group were to make improvements at all levels. The rubric covered the presentation criteria as PSTs and I had agreed upon in previous lessons. The next stage of development of the rubric involved analyzing the work of the middle group to develop the criteria for the Intermediate category, and then I analyzed the work of the lowest group to develop the Emerging category.

The column developed in the rubric based on the Proficient group had the following indicators for each of the quality criteria:

- Introduction/Opening: The learning objective was clearly communicated orally or in written format. The teacher provides an engaging hook to motivate students to become involved in the lesson.
- Connections Made: The teacher makes learning objectives connected to prior learning or life experiences of students.
- Explanations: Explanations are clear and correct. Explanations are connected to the learning objective. The teacher uses more than one representation of concepts and ideas.
- Examples: The examples support the lesson. Examples provide time for student interaction. Examples are relevant to student prior experiences.
- Engagement: Activities are relevant and appropriately challenging. Problems engage students in active participation with hands-on materials and/or technology. Teachers questioning is purposeful and leads students to further inquiry.

- Closure: Teacher provides time for student reflection on lesson content. Closing activity provides formative information for the teacher and an opportunity for teacher feedback.

To continue the rubric development, I analyzed the data results from the middle- rated group and lowest rated group until a full rubric was developed with the indicators created from the performance of the three groups. This fully developed rubric is located in Appendix E.

Interpretations of the Codes

After several cycles of reorganizing and regrouping the initial codes, three major themes emerged from the data: representations of fractions, means of engagement, and conceptual understanding and learning. Students made multiple references to these themes through in student journals and in conversations noted in the teacher observation field notes.

Representation of Fractions

During the learner phase of the intervention, the concepts of fraction representation were the themes of major student epiphanies from the lessons. Students expressed these ideas as a realization that they had just discovered from the Phase 1 lessons. Some of the quotes from student reflections were: “I never realized how many options there were for representing equal values”; “I never thought of trapezoids, hexagons, and triangles as being parts of a whole”; and “I never thought of fractions as pieces.” Before using the concrete manipulatives, some of the students simply saw fractions as numbers to represent amounts used to describe a value. They never had

thought of fractions as parts that could be seen or felt. Students began to use the words “equal shares” and “equal pieces” in communicating their ideas about fractions. They began to be careful to check the size and shape of the whole before representing fractional parts, comparing fractions, or adding and subtracting them. Using the fraction circles helped students to compare the pieces that belong to the same wholes. The pattern blocks helped them to realize that “the whole” is not always in the shape of a circle or a rectangle. Cuisenaire rods helped students to represent different ways of finding equivalent fractions by lining up the different rods. The realizations that occurred to students in this phase of the intervention were made possible by the various representations of fractions used to teach the lessons.

Means of Engagement

The use of concrete manipulatives for a teaching strategy promoted active engagement that encourages exploration and further inquiry. One student wrote, “I have never been more active in learning math than I have been with these lessons.” Students were engaged because they were mostly visual and tactile learners. Four students commented on the fact that manipulatives are visual. One student commented that they helped students to see the whole. “Fraction circles,” remarked one student, “are good for comparing fractions, because you can lay one on top of the other and see the difference.”

Other students commented on the benefit of active engagement with concrete materials for tactile learners. “Physically touching the parts and moving them around helped me to better understand fractions,” wrote one student. Students who were already in field experiences at schools gave opinions of the benefits provided for young tactile

learners. Alison wrote that the students in her field experience class enjoy free play with the manipulatives and create so many different shapes when they are allowed to have this opportunity. “Being familiar with the objects from the free play helps children to use the manipulatives in their designed math lessons,” wrote Alison.

“When I am busy solving a problem with pattern blocks, I am totally immersed in solving the problem without a formula or a procedure. This is new to me, and I enjoy learning through being active and working with my friends,” wrote Cat. Collaboration with others is a key piece of active engagement. Talking about what one is learning and making meaning of it through conversations with others is important in engaging students and in helping them to persevere in solving more challenging problems.

Conceptual Understanding and Learning

“Better than paper and pencil” is a quote from Molly that led me to conclude that some of my students are beginning to value the methods of teaching and learning for conceptual understanding over the traditional memorization of formulas and mimicking of algorithmic steps. One student commented that “seeing is better than memorizing.” Ann wrote, “I like seeing the reasoning behind something.” Ann captured the goal of conceptual teaching methods with this statement. The goal is to understand the reasoning behind a concept so that students can know when and how to use it in problem solving.

Reagan commented that she could do better when she can see a problem in front of her and not just when she reads about it. In my field notes, I had written a quote from a student who wondered why she had been taught through traditional pedagogical methods and not conceptual methods. She asked, “Why were we never taught this way before

now? I finally understand what is going on in math, and I am not just doing stuff without knowing why.” The value of this reformed pedagogy has become evident to these students, and this future class of elementary math teachers is understanding that teaching for conceptual understanding is better than teaching for procedural understanding alone.

Conclusion

The analysis of the data collected from the research study has provided answers to several research questions in the study, but some results are still nebulous. There is evidence to show that strategies using manipulatives such as pattern blocks, fraction circles, and Cuisenaire rods are effective and efficient for developing the beliefs of PSTs that teaching for conceptual understanding is better than teaching for procedural understanding alone.

By using active engagement in collaborative groups, the confidence levels in CCK and SCK math abilities for students with math anxieties increased. This evidence comes from the positive comments made by students in their journal writings and from the increase that occurred in volunteer responses. During the presentations, most students gave the appearance of confidence as they presented their lessons. The strategy of using think–pair–share on a daily basis helped students confirm the validity of their answers before having to share them with the class. This reduced the anxiety levels about class participation. By having students show their reasoning skills on test items, rather than having test items that relied on memorization and procedures, students commented that they felt less anxious about test taking.

Research Question 2 involved finding important factors to consider when developing instructional strategies that promote specialized content knowledge and the intrinsic motivation to teach for conceptual understanding among preservice elementary mathematics teachers? One of the factors I hoped would make a positive difference in intrinsic motivation for PSTS is the understanding of the value of culturally relevant pedagogy. Classroom discourse about the issue was the only strategy used in the limited time of the intervention. Awareness of one's own beliefs about others is the first step in being culturally relevant. Culturally relevant lessons should be beneficial to all students in the class and should link all students with their ancestral and contemporary cultures. Establishing inclusion with regular grouping of students was encouraged in class. I chose strategies that would make class explanations, examples, and activities relevant to students' real-world experiences in the three phases of the intervention. I felt it was important to discuss awareness of learning styles, confidence levels, and any special needs as another strategy for ensuring cultural relevance. From the lesson plans, the evidence gathered showed that students were not effective at planning culturally responsive lessons, but they had an idea of the importance of cultural relevance and made efforts to plan for it. Most lesson plans involved superficial lessons that included menus, clothing, or holidays in other cultures.

The instructional strategies that were effective and efficient to best develop SCK for preservice teachers involved conceptual teaching methods using concrete materials and multiple representations of concepts. Other effective strategies included using a constructivist approach to having students make their own meaning from their learning.

Phase 2 of the intervention was effective in getting students to identify criteria for quality teaching and then evaluating lessons based on their self-created instrument.

From the study, a conclusion can be made that the best performing group for developed SCK had the most CCK. The conclusion from the data of this study leads me to believe that the two are not mutually exclusive events and that one leads to the other. Although there is a correlation between the two types of knowledge, it cannot be inferred that the relationship is causal.

In Chapter 5, I outlined the recommendations for using these findings for future teaching practices and suggestions for future research.

Chapter 5

Discussions, Conclusions, and Recommendations

This qualitative case study was conducted to assess the effectiveness of strategies used to develop the conceptual understanding of math concepts for preservice teachers and to increase their conceptual content mastery, as well as the specialized content mastery that they will need to feel empowered to teach their future students. The study focused on theories of reformed pedagogy (Smith, 2013), MKT categories of CCK and SCK (Ball et al., 2008), and culturally responsive teaching (Ladson-Billings, 2009). The goal of this study, and a crucial aspect of PSTs' future teacher practices, was supporting PSTs to develop a professional disposition to become confident and continuous learners with a belief in the efficacy of conceptual methods of teaching mathematics.

The future quality of the education that students in my state and in the nation receive depends on providing quality PST education programs in our colleges and universities. This action research study has the potential to impact the quality of math educators who will teach underserved and diverse populations of students, who need them most. Providing PSTs with the necessary tools from the theoretical framework of this study, which include CCK, SCK, reformed pedagogy, and culturally responsive teaching, can help decrease the achievement gap in areas of poverty and among different populations of students (Ball et al., 2008; Ladson-Billings, 2009; Smith, 2013).

I enacted several instructional strategies in an attempt to increase the common content knowledge (CCK) and the SCK of PSTs. I measured the enactment of these strategies, the impact of these strategies on the intrinsic motivation of PSTs to value conceptual mathematical understanding over a procedural understanding of mathematics, and the PSTs' motivation to teach for conceptual mathematics understanding after the intervention and proved them to be effective strategies for the majority of the PST participants, who showed significant development in their CCK after the implementation of the research intervention. The development of SCK was evident from the implementation of Phase 2, in which PSTs were able to define and evaluate quality criteria for teaching and participate in the development of a rubric to assess their own work. During the third phase of the intervention, some students were able to demonstrate their knowledge of how to use the SCK in practice, but this was not the case for many students. This was a weak point in the intervention, since PSTs could discuss and identify quality teaching practices but were not completely proficient in modeling the practices themselves.

I designed the intervention from the perspective that quality math teacher education should provide opportunities for PSTs to develop both conceptual and specialized understanding of the content they will teach and become familiar with instructional strategies that will foster equitable learning for all students (Ball, 2003). The primary research question that guided the data collection was:

1. What are the important factors to consider when developing instructional strategies that promote specialized content knowledge and the intrinsic motivation

to teach for conceptual understanding among preservice elementary mathematics teachers?

Reflections and Implications

This action research study is clearly related to the findings in the literature of Chapter 2. The literature provided justifications for the need of research in supporting PSTs in developing mathematical skills to improve their future practices. Evidence also suggests that the theoretical framework on which this study is based is sound and effective pedagogy that quality teachers should experience and internalize.

The literature first provided evidence of a PST knowledge gap and suggested that elementary teachers in the United States differ in their understanding of the mathematics they teach (Hill, 2010). In 2000, the National Council of Teachers of Mathematics (NCTM, 2000) found that the big ideas of mathematical knowledge were not taught in the average PST course, and therefore, teachers could not teach what they had not been trained to do (Ball, 1993). Due to the gap in PST knowledge of math, many teachers relied on textbooks for explanations, examples, and answers. The lack of conceptual understanding forced these teachers to become attached to procedures (da Ponte & Chapman, 2008) and to pass this limited knowledge of conceptual understanding along to their math students.

The limited knowledge of U.S. public school students is often reflected in international, national, and state standardized test scores and causes legislators to ask for an overhaul in education that includes curriculum and standards (Ball, 1993). These changes produced little positive change in the problem of the math deficiencies in

American public schools (Ball, 1993), and Ball suggested that the focus should shift to the teaching methods for how the curriculums and standards are taught. Cipri (1992) stated that PST programs in colleges and universities were the best places to attack the problem, since this is where teachers learn methods for teaching. Ball (1993) agreed on the setting for developing these skills in PST and claimed that MKT would produce high quality teaching. Research shows that math anxiety presents a challenge for elementary school teachers who usually have a lower math content knowledge and higher math anxiety than average college students (Novack & Tassell, 2017). Years later the problem still exists. and Beckmann (2014) echoed the sentiments of Ball and agreed that PSTs need to know the *why* as well as the *how* of solving problems in math and be able to provide relevant applications. The AMTE (2016) emphasizes that well prepared beginning teachers have a positive disposition toward the mathematics that they teach.

The data for my pretest for CCK and survey of attitudes and beliefs in mathematics showed evidence to confirm the findings from the literature. Before the case study intervention process, only 29% of my PSTs were able to score a passing grade of 75 on a four-question fraction operations test. The attitudes and beliefs survey showed a 2:1 to one ratio for negative responses to the statement, “I feel confident in my math abilities.” Four questions regarding math anxieties produced even higher negative responses, with 87% of the class agreeing that they had math test anxiety issues, and 67% of the class agreeing that they had anxiety about responding aloud in a math class. These findings are consistent with those of past researchers, who have observed the same problem areas that require support strategies for PSTs in elementary math programs.

In an effort to increase the CCK of PSTs, I designed the first phase of the three-phase intervention plan using strategies that promote conceptual understanding. During this phase of the intervention, I taught the lessons to PSTs in the same manner that they should use to teach their future students. Creating multiple representations for fractions using a variety of manipulatives led to discussions that helped in the development of number sense and understanding of concepts (Thanheiser et al., 2010). By using these methods, the CCK and SCK of PSTs can be increased through solving the problem (CCK) and explaining the problem through multiple representations (SCK) (Thanheiser et al., 2010).

The results of Phase 1 showed that strategies of conceptual understanding using multiple representations and a variety of manipulatives were effective in increasing the CCK of the PSTs in my class for all four basic arithmetic operations on fractions. The most significant gains were made in CCK of multiplication and division, the two areas for which the PSTs were tasked with planning and implementing a lesson. Whether the increase in CCK came because of strategies used in Stage 1 or the task of planning and implementing a lesson in Stage 3 is not clear.

A better conclusion as to the source of understanding could have been more obvious if the posttest had been given directly after Stage 1 of the intervention, rather than waiting until the end of the entire intervention process to assess the growth of CCK. Also, I would have been aware of the additional support that some of the PSTs needed in gaining CCK if the posttest had been given earlier. This would have caused a modification in my intervention plan and an extension in my timeline, but a change that

would have been more beneficial to the intervention outcome than it would have been a detriment.

For the pretest and posttest questions, I believe that I would have gotten better information from having four or five questions that were simply computations of fractions and then four or five that were written in word problem format. Since the questions were in word problem format for this intervention, it is unclear if incorrect answers indicated that students lacked the ability to perform the procedures, or if they were lacked the ability to identify the proper operation to use in order to solve the problem. One likely cause for the poor performance by some PSTs on the two tests could possibly be related to the word problem format in which they were presented.

Successful solutions to word problems require that the student not only be able to read and know the meaning of words, but they must be able to integrate the word meanings into the more complex task of identifying the problem type. This requires a knowledge of both linguistics and math, and an ability to effectively combine the two. Math anxiety, which was evident among my preservice teachers, was a likely culprit for the inability of lower performing students to combine their knowledge of linguistics and math. Through the strategy of translating the word problems into models or diagrams, I hoped to help PSTs to be more successful in approaching word problems. It is evident that many PSTs did not develop the skills of translating word problems into other formats, which would have helped them decrease their anxiety and be more successful in solving the problems.

The implications for future math education in our state and nation is that there are potentially PSTs with the inability to teach math effectively who are graduating from our preservice program. It is possible that PSTs can pass the course despite their inability to demonstrate adequate CCK and/or SCK. This has been and remains to be an ongoing and contentious issue among my colleagues. It has been my policy to place enough of the weighted grade on the CCK so that a PST cannot score higher than a “C” in the course. This “C” is the required passing score to continue in the course sequence. With any other deficiencies in the class assignments, the PST would not be successful and would have to repeat the course. Is this enough of a safeguard against have graduates enter the teaching workforce who are not prepared?

The PSTs in this course have already passed Praxis and are successful in many other areas of teaching and in other math assignments in the class. Should they fail the math class because of the inability to demonstrate the CCK of some of the course concepts? Although the intervention focused on fractions, the entire course focused on all rational numbers. Fractions were the most challenging concept for the PSTs.

All math teacher preparation programs must reflect on strategies for strengthening the mathematics course requirements for elementary teachers. The content of the courses, the requirements for successful completion, and the assessment of PST readiness must be considered. Without careful attention to highly effective teacher programs, many school districts will be forced to hire ineffective teachers who will not deliver equitable education to our children. For those in high poverty districts, this will only perpetuate the cycle of poverty in their lives.

At the end of Stage 2, student responses in class, in journals, and on exit slips showed evidence of increased SCK in preservice participants. The initial intervention lesson in this stage was for students to create a list of quality criteria for teaching, and students responded with mature answers that were consistent with the ideas of Ball, Thames, and Phelps (2008). The gallery walk and the whole class discussion that followed led to the development of a student made observation checklist that PSTs used for the remainder of the intervention to evaluate the lessons of others. Comments from the evaluations showed growth in understanding of quality teaching criteria.

I embedded reformed pedagogy (Smith, 2013) in all aspects of the intervention process. The reformed pedagogy ideas are not new, and the roots can be traced back to John Dewey, who believed that children should learn by doing and that they should make meaning and construct their own knowledge as a result of their own experiences (Schiro, 2013). Active engagement is a part of the reformed pedagogy and is crucial for developing deep conceptual understanding in math (Piaget, 1973). This constructivist idea has not gone without challenge, as opponents point to the failure of American children to learn basic skills. The “back to basics” movement versus the constructivist approach has been a point of contention for years (Klein, 2003), but the recent efforts by the National Council of Teachers of Mathematics (2000) to push the reform has made it shift once more.

During Stage 1 of the research study intervention, the idea of using hands-on constructivist activities and problem solving were an integral part of the lessons. In all

stages, the idea of cooperative learning was implemented and proved to be successful in developing both CCK and SCK, as is evidenced in the lesson planning comments in Stage 3 that showed mature thinking consistent with the ideas of quality teaching criteria developed by Ball (1993).

The literature also addressed the benefits of using culturally responsive teaching practices to advocate for responding to cultural differences between teachers and students (Ladson-Billings, 2009). Strategies planned to support culturally responsive pedagogy included building respectful listening and speaking habits, developing respect for ideas and differences in others, and learning to make lessons meaningful by drawing on prior knowledge and experience of students (Wlodkowski & Ginsberg, 1995).

Opportunities to practice these culturally responsive techniques were given in each phase of the intervention, as collaborative engagement, whole group discourse, and constructive critique were an essential part of communicating understanding of mathematical ideas. Students participated in discussions on modifying lessons to meet the needs of diverse student populations and different ability groups during Phases 2 and 3 of the intervention. Final lesson plans that students submitted in Stage 3 of the intervention showed mostly superficial answers that did not show evidence of a good understanding for modifying lessons to meet individual student needs. In hindsight, I can see the need for spending more time on strategies of meeting the needs of all students.

Lisa Delpit (2012) explained that children of high poverty families are often adept at problem solving due to the family situations and the roles that they must play in running the household, caring for siblings, and solving their own problems independent

of any adult help. It is ironic that these same children are relegated to remedial basic skills classes to work on drill and practice worksheets to raise their test scores, when this is totally incompatible with their strengths, their prior experiences, and learning styles, and is dooming these children to academic failure (Delpit, 2012). This important aspect of teaching was lost in the research study due to the time constraints that I had put on my intervention plan. I focused more on the other theories that addressed the needs of PSTs to become high quality math teachers and did not allocate enough time to effectively teach them how to choose relevant and meaningful math activities and instruction to meet the culturally diverse needs of students.

I do, however, believe that my successful PSTs are prepared to address the issues of equity in high poverty areas, since they possess the CCK, SCK, and intrinsic motivation to teach with the reformed pedagogical strategies necessary to the needs and strengths of these students. AMTE (2016) states, “By ensuring that those who complete teacher preparation programs have strong content knowledge, understanding of the practice of mathematics, and positive mathematics identities, programs are promoting a teaching workforce that provides an equitable education for all students” (p.38).

In light of these findings for my research study, I can conclude that PSTs need more support in creating meaningful and relevant lessons for diverse groups, and that the implications of giving more focus to this need can potentially have a larger impact on the teacher quality in our state. South Carolina has high poverty districts throughout the state, particularly along the infamous I-95 “corridor of shame.” High quality teachers are needed most in these areas, but the poorer working conditions and the lower salaries often entice these teachers to work in the more affluent districts.

The needs of individuals must also be addressed in gender differences as well. The literature shows that girls often have poor concepts of their own abilities in math (Bell & Norwood, 2007) and often suffer math anxiety that interferes with their abilities to perform math computations and problem solve (Richardson & Suinn, 1972). These findings from review of the literature were evidenced in my own research study. The confidence levels of my own PSTs were very low before the intervention, and anxiety levels were very high.

Strategies to build the confidence in my PSTs' math abilities and to decrease their math anxieties included using hands-on learning in cooperative groups (Clewel, 1987), giving students input and choices on assessments and assignments, teaching with relevant and meaningful problem solving, and using multiple methods and representations (Hanson, 1992). Student reflections in their lesson planning assignment showed significant improvements in attitudes of professional disposition and confidence in abilities to teach mathematics to future learners.

One of the most significant results of my findings was that by developing an inductive rubric from the performance indicators of my best group's presentation, I was able to improve my own SCK. I used the rubric development as a tool to assess the student performances and to give feedback to students. By using the best performing group as a model to write indicators for a proficient column, I was able to use the information as formative information for my instruction. To help PSTs to strive for a higher degree of quality teaching, I used the indicators for the proficient column to make decisions on how they could improve their teaching. These ideas formed the basis for writing the indicators for an exemplary column.

From this inductive rubric development, I saw a strategy for improving my own practice and discovered the recursive relationship that existed in the inductive rubric writing process. A rule is recursive if it is such that it can be applied to its own output an indefinite number of times, yielding a total output that is potentially infinite (Hauser, Chomsky, & Fitch 2002). In this case, I applied my SCK to support the SCK of PSTs in my class. By using the rubric based on their SCK to form my next instruction, I am using their SCK to develop my SCK, which I will then use to teach SCK, and will repeat indefinitely. I based the other columns for the inductive rubric on the performances of the middle group (Intermediate column) and lower group (Emerging column). Although I wrote the indicators based on these groups, some student groups did cross back and forth between columns for different criteria.

Reflection on Action Research

Action research is done in collaboration with practitioners or community members (Herr & Anderson, 2015) to improve their own practice. The research process is oriented to an action that will address a particular problematic situation (Herr & Anderson, 2015) and is collaborative (Kemmis & McTaggart, 1987). Action research demands an intervention and constitutes a cycle of developing a plan, implementing a plan, observing, reflecting, and repeating (Herr & Anderson, 2015).

This action research study has all the characteristics of action research. The purpose of the study was to develop ways to improve my practices by supporting the PSTs who I teach to increase their CCK and SCK and to develop professional dispositions to support them in being confident, continual learners. This particular

problem situation is that the number of highly qualified teachers with conceptual understanding of mathematics is limited in our state's and nation's schools, particularly in high poverty areas. I have been in collaboration with colleagues to help me interpret data and coding and to help me to recognize any personal bias toward research results. I have used multiple sources of data and used triangulation of the data to write a rich, description of the data interpretation. I have also used member checking with my PST participants to ensure the accuracy of my findings. Using these three methods, the research study meets the validity requirements of action research.

The three-stage intervention plan for the study satisfies the demand for intervention in action research. The plan is cyclical in nature, as were the surprising findings from my inductive rubric development. The case study design was a perfect fit for the study because it was an in-depth analysis of my PST program, in which I collected information over a sustained period.

Transferability

Transferability is synonymous in qualitative research with generalizability. It is established by providing evidence that the research study could be applicable to other contexts or populations (Creswell, 2014). Reliability and validity in qualitative studies are more easily confirmed in qualitative studies. Reliability indicates that the researcher's approach is consistent across all tests of what it measures (Mills, 2007). A check for reliability can be done in a qualitative study by using colleagues to help with cross checking of coding (Creswell, 2014). Validity is used to demonstrate accuracy of findings

and can be verified by member checking, triangulation of data sources, and peer debriefing.

My research study meets standards of validity and reliability as previously described, but without repeated studies of additional cases, one can only conclude that this case study and its findings apply only to a particular study situation and cannot be generalized to other studies.

Limitations and Suggestions for Future Research

This action research study was limited to a focus on operations of fractions. Further research is needed on using these theories and strategies to be able to transfer these results to other branches of mathematics, such as geometry, or to other grade levels of mathematics, such as increasing the effectiveness of secondary PSTs to teach for conceptual understanding. My prediction is that using these strategies and theories would be effective in teaching mathematics in other strands of math and at different grade levels, but this study does not allow that conclusion to be drawn. Although the specific topic of focus limited the conclusions that can be made, it does leave room for further research to be done in these other areas. The implications of this study are potentially useful for other college professors or clinical professors at other teaching universities and may also lead to future research ideas using the same intervention design.

I would like for my next research study to focus on geometry, another area of mathematics that, in my experience, PSTs find intimidating. Modifications to the research design would include administering the posttest for CCK immediately after the first intervention phase, rather than after the completion of the intervention. This would ensure

me that the results were influenced by Phase 1 and not a combination of other phases of the intervention. It would also give me data that would inform future instruction to better support students still lacking in CCK. I would also make the intervention phases last for longer periods of time to allow for more attention to making math lessons more culturally responsive and for allowing more time for students to prepare and make adjustments for their presentations.

Other areas of research interest are strategies for teaching social justice in math, an area that is often viewed as a neutral discipline. Teaching PSTs how to weave social justice themes into math lessons without superficially forcing the math issue would be a valuable tool for PSTs to have going into high poverty areas to teach, and it would possibly give PSTs the confidence and intrinsic motivation to want to teach in these areas. The impact of this research could be potentially huge for the state of South Carolina and its underserved areas, particularly along the I-95 “corridor of shame.”

Summary

The findings of this research study confirm that students gain conceptual understanding from using manipulatives such as Cuisenaire rods, pattern blocks, and fraction circles. Students made comments that made me aware of the importance that they place on these learning activities and of the importance of cooperative learning strategies in increasing the confidence in their personal math abilities. They also made comments as to the value of the think–pair–share method as a tool for communicating and reflecting on mathematical ideas. PSTs confirmed that they would use these strategies in their own classrooms to promote confidence in mathematics discourse and learning. The habit of

sharing ideas and materials with others was suggested to PSTs as a habit that they might want to continue with colleagues to further promote positive professional dispositions and to further increase SCK. Through the journal writings and class discourse, there was clear evidence to show that PSTs valued the methods of reformed pedagogy and preferred them to traditional methods.

The strategies that were used in the research study will continue to be a part of my teaching strategies for PST classes, but I will also continue to research and keep updated for current and relevant ideas in problem solving and meaningful experiences for PSTs. This study has made me more aware of the implications of supporting PSTs to teach in high poverty areas and to encourage them to be intrinsically motivated to teach in these areas. My classes will have a wider focus on planning math lessons around social justice issues when the opportunity is present.

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Appendix A
Informed Consent Form

Dear Student,

I am a doctoral candidate at the University of South Carolina. I am conducting research as part of the requirements for my degree in Curriculum and Instruction, and I would like to invite you to participate. If you decide to participate the results of your survey, classwork discussions, formative assessments, and teaching segments will be included in the research data information. Although we have discussed the general nature of your tasks, the full purpose of the study cannot be explained because doing so would bias the study results.

Participation is confidential and anonymous. At no time will your name or any other identifying information be used in reporting results. The results of the study may be presented or published, but your identity will not be revealed.

Participation, non-participation, or withdrawal from the study will in no way affect your grade. I will be happy to answer any questions you have about the study.

Jane Wilkes

I agree to participate in this study. I have been given a copy of this form for my own records.

If you wish to participate, you should sign below.

Signature of Subject / Participant

Date

Signature of Qualified Person Obtaining Consent

Date

Appendix B
Pre/Posttest for CCK

Questions for CCK

Name _____

Please fully explain your reasoning in solving each problem.

1. Plans for a new park show that $\frac{3}{5}$ of the park will be for a playground. Of the designated playground area, $\frac{1}{4}$ will be reserved for special needs children and families. What fraction of the total new park will be a playground for special needs children and families?

2. Mr. Smith's will provides that his five children will share his estate according to the following provisions. Al receives $\frac{1}{3}$ of the inheritance, Bob receives $\frac{1}{4}$, Cal receives $\frac{1}{5}$ and Don receives $\frac{1}{6}$. What fractional part of Mr. Smith's estate does Ed receive?

3. Which is the larger of the two fractions: $\frac{995}{8432}$ or $\frac{995}{8429}$?

4. Jane is making apple turnovers. If she uses $\frac{3}{4}$ of an apple for each turnover, how many turnovers can she make with 18 apples?

Appendix C

Attitudes Toward Mathematics Teaching and Learning

Answer each question about your beliefs and feelings as a learner by circling the best choice.

1. I enjoy math and feel confident in my math abilities.

Strongly Agree Agree Neutral Disagree Strongly Disagree

2. I get nervous before math tests.

Strongly Agree Agree Neutral Disagree Strongly Disagree

3. I draw a blank during math tests, even though I feel well prepared beforehand.

Strongly Agree Agree Neutral Disagree Strongly Disagree

4. I am anxious when I feel that the teacher is going to call on me.

Strongly Agree Agree Neutral Disagree Strongly Disagree

5. I think boys are better at math than girls.

Strongly Agree Agree Neutral Disagree Strongly Disagree

6. I am comfortable answering questions in math class.

Strongly Agree Agree Neutral Disagree Strongly Disagree

Appendix D
Evaluation Checklist

Teacher Behavior	Observed	Evidence/ Comments
Lesson objective clearly defined		
Instruction matches objective		
Explanations are clear and accurate		
Multiple examples to reinforce explanations		
Variety of teaching techniques used		
Relevancy of lesson is obvious or discussed		
Students engaged in learning		

Teacher makes connections to prior learning or prior experiences		
Teacher recognizes/anticipates student errors and misconceptions		
Appropriate feedback given		
Talk time balanced between students and teacher		
Lesson well organized in logical sequence		
Higher order questioning used		
Teacher encouragement and verbal praise given		
Closure of lesson		

Appendix E

Inductive Rubric

	Exemplary	Proficient	Intermediate	Emerging
Introduction/ Opening	The learning objective was clearly communicated orally and in written format. The teacher provides an engaging and challenging hook to motivate students to become involved in explorations of the lesson concepts.	The learning objective was clearly communicated orally or in written format. The teacher provides an engaging hook to motivate students to become involved in the lesson.	The learning objective was communicated orally or in written format. The teacher provides a hook to motivate students to become involved in the lesson.	The learning objective was communicated orally or in written format. The teacher did not provide a hook to motivate students to become involved in the lesson.
Connections	The teacher makes learning objectives connected to prior learning, personal life experiences of students, and other disciplines.	The teacher makes learning objectives connected to prior learning or life experiences of students.	The teacher makes a weak connection with the learning objectives and prior learning or life experiences of students.	The teacher makes no connection between learning objectives and prior learning or life experiences of students.
Explanations	Explanations are clear,	Explanations are clear and correct.	Explanations are not completely	Explanations are not clear and

	correct, and generate inquiry. Explanations are clearly connected to the learning objective and connections to the objective are verbalized. The teacher uses multiple representation of concepts and ideas.	Explanations are connected to the learning objective. The teacher uses more than one representation of concepts and ideas.	clear and some errors are made, but corrected. Explanations are somewhat connected to the learning objective. The teacher uses one representation of concepts and ideas.	errors are observed. Explanations are not fully connected to the learning objective. The teacher uses one or no representations of concepts and ideas.
Examples	The examples clearly support the lesson. Examples provide time for student interaction and sharing of opinions. Examples are relevant to student prior experiences and induce curiosity.	The examples support the lesson. Examples provide time for student interaction. Examples are relevant to student prior experiences.	The examples somewhat support the lesson. Examples provide time little to no for student interaction. Examples are relevant to student prior experiences.	The examples do not support the lesson. Examples provide no time for student interaction. Examples are not relevant to student prior experiences.
Engagement of Students	Activities are personally meaningful, relevant and appropriately challenging. Problems engage students in active	Activities are relevant and appropriately challenging. Problems engage students in active participation with hands-on materials and/or technology.	Activities are somewhat relevant but not appropriately challenging. Problems do not fully engage students in active participation with hands-on	Activities are not meaningful to the lesson objective. Problems do not engage students in active participation with hands-on materials or technology.

	participation with hands-on materials and/or technology. Teacher questioning is purposeful and leads students to further inquiry, curiosity, and exploration.	Teacher questioning is purposeful and leads students to further inquiry.	materials and/or technology. Teacher questioning is based on lower taxonomy levels.	Teacher questioning is not purposeful or not evident.
Closure	Teacher provides time for student reflection on lesson content. Closing activity provides valuable formative information for the teacher and an opportunity for teacher feedback.	Teacher provides time for student reflection on lesson content. Closing activity provides some formative information for the teacher and an opportunity for teacher feedback.	Teacher provides time for student reflection on lesson content. Closing activity provides little formative information for the teacher and few opportunities for teacher feedback.	Teacher provides no time for student reflection on lesson content. Closing activity provides no formative information for the teacher and no opportunity for teacher feedback.